Autumn Block 5

Multiplication and division B

Small steps

Step 1	Multiply a 2-digit number by a 2-digit number
Step 2	Multiply up to a 4-digit number by a 2-digit number
Step 3	Solve problems with multiplication
Step 4	Short division
Step 5	Divide a 4-digit number by a 1-digit number
Step 6	Division using factors
Step 7	Introduction to long division
Step 8	Long division with remainders

Small steps

Step 9	Solve problems with division
Step 10	Efficient division
Step 11	Solve multi-step problems
Step 12	Order of operations
Step 13	Mental calculations and estimation
Step 14	Reason from known facts

Multiply a 2-digit number by a 2-digit number

Notes and guidance

In this small step, Year 5 children are introduced to multiplying a 2-digit number by a 2-digit number for the first time.

Start by recapping the formal written method for multiplying a number with up to four digits by a 1-digit number. As children progress to multiplying two 2-digit numbers, an area model may be used to develop their understanding before using the formal written method.

Year 5 children will need to spend longer looking at how to use the formal written method and discuss the role of the zero placeholder in the ones column when multiplying by the tens digit. Year 6 children could focus more on problem solving and consider efficient methods.

Things to look out for

- Children may omit the zero as a placeholder when multiplying by the tens digit.
- When an exchange is needed in the multiplication steps, children may accidently also add the exchanged number in the final addition. Crossing out the exchange once it has been used may help to prevent this.

Key questions

- What are you multiplying by first?
- Why is there a zero in the ones column when multiplying by _____?

Possible sentence stems

- First, I multiply _____ by _____ ones.
  Then I multiply _____ by _____ tens.
  Finally, I add together _____ and _____

Single age small step links

- Multiply up to a 4-digit number by a 1-digit number (Y5)
- Multiply a 2-digit number by a 2-digit number (Y5)

- N/A

National Curriculum links

- Multiply numbers up to four digits by a 1- or 2-digit number using a formal written method, including long multiplication for 2-digit numbers (Y5)
- Multiply multi-digit numbers up to four digits by a 2-digit whole number using the formal written method of long multiplication (Y6)

Multiply a 2-digit number by a 2-digit number

Key learning

- A theatre has 2,353 seats.

  A show at the theatre is sold out for 3 nights.

  How many people go to see the show in total?

Th	H	T	O
1,000 1,000	100 100 100	10 10 10 10 10	1 1 1
1,000 1,000	100 100 100	10 10 10 10 10	1 1 1
1,000 1,000	100 100 100	10 10 10 10 10	1 1 1

		2	3	5	3	
	×				3	

- Tom and Esther are working out 42 × 21

Tom

×	20	1
40	800	40
2	40	2

800 + 40 + 40 + 2 = 882

(42 × 1)
(42 × 20)

Esther

			4	2	
	×		2	1	
			4	2	
		8	4	0	
		8	8	2	

What is the same and what is different about Tom and Esther's methods?

- Complete the calculation to work out 43 × 25

			4	3	
	×		2	5	
		2	1 (1)	5	
		8	6	0	

(43 × 5)
(43 × 20)

Use this method to work out the multiplications.

24 × 36	68 × 23	74 × 46

- Work out the multiplications.

52 × 21	52 × 22	51 × 22

What is the same and what is different about the answers?

Could you have worked any of them out in a different way?

- Miss Lee completes 35 press-ups every night in March.

  How many press-ups does she complete in total?

Multiply a 2-digit number by a 2-digit number

Reasoning and problem solving

Max is multiplying 56 by 24

			5	6	
	×		2	4	
		2	2_2	4	
		1	1_1	2	
		3	3	6	

What mistake has Max made?

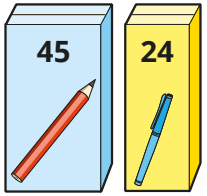
What is the correct answer?

Max has forgotten to use zero as a placeholder when multiplying by 2 tens.

1,344

Pencils are sold in boxes of 45

Pens are sold in boxes of 24

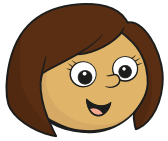


Mrs Morgan buys 21 boxes of pencils and 42 boxes of pens.

How many more pens does she buy than pencils?

63

Kim is working out 48 × 73



48 × 73 = 480

Use estimation to show that Kim must be incorrect.

What is the correct answer?

multiple possible answers, e.g.

50 × 70 = 3,500

3,504

Multiply up to a 4-digit number by a 2-digit number

Notes and guidance

In this small step, children now multiply numbers with up to four digits by 2-digit numbers.

Children must be confident with multiplying both 2- and 3-digit numbers by a 2-digit number, before moving on to multiplying a 4-digit number. As they are now working with greater numbers, it is important that they understand the steps in the long multiplication method. An area model using place value counters could support children who need it, but the emphasis should be on using the formal written method.

As with the previous step, children need to understand the role of zero in the ones column when multiplying by the tens.

Year 6 children could also solve word problems and/or multi-step problems.

Things to look out for

- Children may omit the zero as a placeholder when multiplying by the tens digit.
- Children may forget to include exchanges in their calculations.
- Children need to be secure with their times-tables or have strategies for deriving them.

Key questions

- What number do you multiply by first?
- When do you need to make an exchange? How do you do this?

Possible sentence stems

- First, I multiply _____ by _____ ones.
  Then I multiply _____ by _____ tens.
  Finally, I add together _____ and _____

Single age small step links

- Multiply a 3-digit number by a 2-digit number (Y5)
- Multiply a 4-digit number by a 2-digit number (Y5)

- Multiply up to a 4-digit number by a 2-digit number (Y6)

National Curriculum links

- Multiply numbers up to four digits by a 1- or 2-digit number using a formal written method, including long multiplication for 2-digit numbers (Y5)
- Multiply multi-digit numbers up to four digits by a 2-digit whole number using the formal written method of long multiplication (Y6)

Multiply up to a 4-digit number by a 2-digit number

Key learning

- Complete the calculation to work out 214 × 32

			2	1	4	
	×			3	2	
			4	2	8	
		6	4_1	2	0	

(214 × 2)

(214 × 30)

Use this method to work out the multiplications.

213 × 13	452 × 21	562 × 43

- Complete the calculations.

			2	3	2	4	
	×				2	1	
			2	3	2	4	
		4	6	4	8	0	

(2,324 × _____)

(2,324 × _____)

			5	3	4	6	
	×				4	3	

(_____ × _____)

(_____ × _____)

- Work out the multiplications.

4,513 × 43	6,153 × 68	9,748 × 74

- Estimate the answers to the multiplications.

3,128 × 41	5,981 × 89	8,492 × 99

Work out the multiplications.

How close were your estimates to the actual answers?

- Write <, > or = to compare the calculations.

3,571 × 46 ◯ 3,653 × 44

3,571 × 45 ◯ 3,653 × 44

3,571 × 45 ◯ 3,653 × 43

Did you need to work out the calculations each time?

Multiply up to a 4-digit number by a 2-digit number

Reasoning and problem solving

Huan has done some calculations.

			4	3	5	6	
	×				3	1	
			4	3	5	6	
		1_1	3_1	0_1	6_1	8	
		1	7	4	2	4	
				1	1		

				5	2	9	6	
	×					7	3	
			1	5	6	7	8	
		3	5	4	3	2	0	
		3	6	9	9	9	8	

What mistakes has Huan made?

Work out the correct answers.

135,036

386,608

2	3	4
5	6	7

Write the digits in the boxes to find both the greatest and the smallest product.

You can use each digit only once each time.

	×							

greatest product:

7,432 × 65 = 483,080

smallest product:

3,567 × 24 = 85,608

Solve problems with multiplication

Notes and guidance

In this small step, children apply their knowledge of multiplication to solve problems. They practise both the formal written method for multiplication and the use of efficient mental strategies.

It is important that children explore a variety of methods to solve multiplication problems and discuss which is the most efficient. They may refer to known facts to help them derive unknown facts. For example, to calculate 9,999 × 7, they can work out 10,000 × 7 and then subtract 1 lot of 7

Children explore using factors to find the answers to multiplication problems, such as multiplying by 5 and then by 7 as an alternative to multiplying by 35. This is a useful strategy for children who have sufficient times-table knowledge but make errors with the algorithm for long multiplication.

Things to look out for

- Children may try to use formal methods when alternative strategies would be more appropriate.
- Children may need support to identify the most efficient method; for example × 100 and then subtract × 1 may be better than × 90 and then add × 9

Key questions

- Why can you multiply the numbers in any order?
- What strategy can you use to solve this problem?
- Is there a more efficient method?

Possible sentence stems

- To multiply by _____, I can multiply by _____ and add/subtract _____ to/from the product.

Single age small step links

- Solve problems with multiplication (Y5)
- Solve problems with multiplication (Y6)

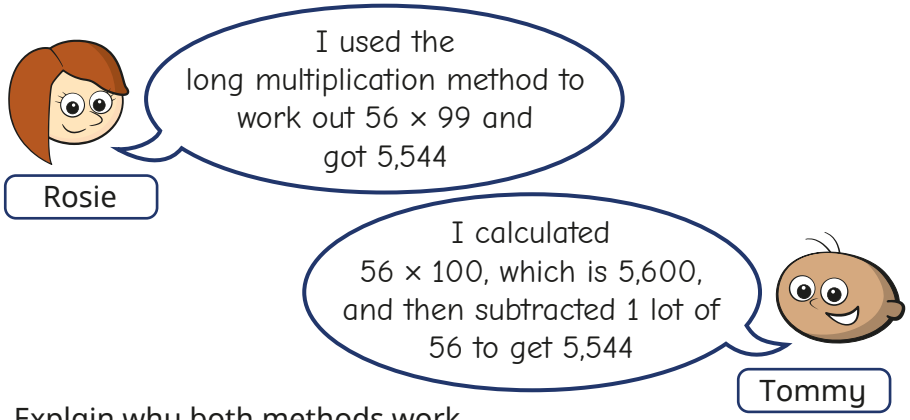
National Curriculum links

- Solve problems involving multiplication and division, including using their knowledge of factors and multiples, squares and cubes (Y5)
- Perform mental calculations, including with mixed operations and large numbers (Y6)
- Solve problems involving addition, subtraction, multiplication and division (Y6)

Solve problems with multiplication

Key learning

- Rosie and Tommy have worked out 56 × 99



  Explain why both methods work.

  Whose method do you prefer? Why?

  Use your preferred method to work out the multiplications.

  43 × 101 | 538 × 99 | 502 × 48

- Here is a strategy for multiplying numbers by 5

  Multiply the number by 10 and find half of the answer.

  Use the strategy to work out the multiplications.

  76 × 5 | 714 × 5 | 6,034 × 5 | 8,428 × 5

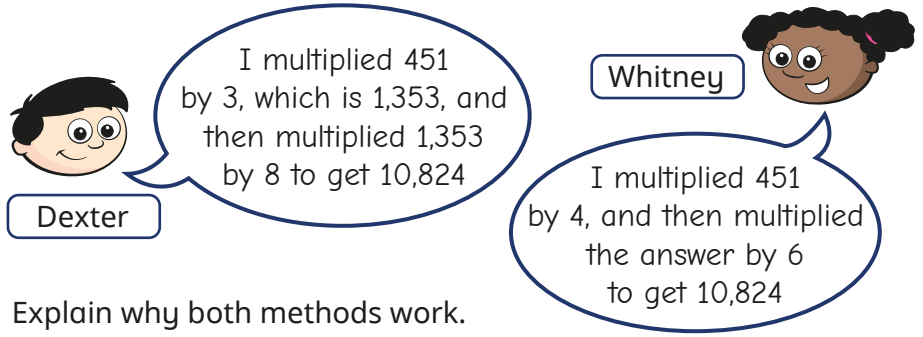
  Why does the strategy work?

- Explain why 93 × 4 = 94 × 2 × 2

  Find the missing numbers.

  - 78 × 14 = 78 × 2 × _____
  - 525 × 25 = 525 × 5 × _____
  - 438 × 15 = 438 × _____ × _____

- Dexter and Whitney have worked out 451 × 24



  Explain why both methods work.

  Whose method do you prefer? Why?

  Use factor pairs to work out the multiplications.

  25 × 516 | 742 × 18 | 814 × 32

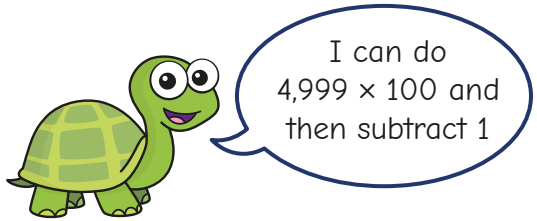
- A supermarket sells 1,214 apples per day.

  How many apples will it sell in 4 weeks?

Solve problems with multiplication

Reasoning and problem solving

Tiny is working out 4,999 × 99



Is Tiny correct?

Explain your answer.

No

FLIGHTS TO JAPAN	
Economy	£1,799
First Class	£3,497

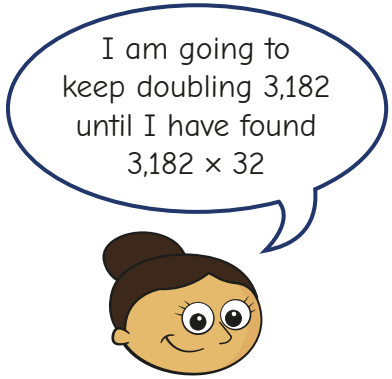
On a flight to Japan, there are 98 economy seats and 24 first class seats.

Tickets for all the seats are sold.

How much money does the airline receive from ticket sales?

£260,230

Dora is working out 3,182 × 32



How many calculations will Dora have to do?

Use Dora's method to work out 3,182 × 32

How else could Dora multiply by 32?

Talk about it with a partner.

five calculations

3,182 × 2 = 6,364

3,182 × 4 = 12,728

3,182 × 8 = 25,456

3,182 × 16 = 50,912

3,182 × 32 = 101,824

Short division

Notes and guidance

Building on informal methods used in previous years, this small step introduces Year 5 children to the formal written method of short division. They use the formal method to divide a 2-digit or 3-digit number by a 1-digit number, initially without an exchange and then with an exchange.

The formal calculation is shown alongside familiar models, in particular part-whole models, place value counters and place value charts. In this way, the structure of short division becomes clear, enabling children to see the relationship between the model and the formal written method.

Children may need to list multiples of the number they are dividing by to help them if their times-table knowledge is not secure. Dividing 4-digit numbers is covered in the next step, including calculations involving remainders.

Things to look out for

- Children may work from right to left, as with addition, subtraction and multiplication.
- When dividing numbers that include zeros as placeholders, children may make errors with place value.

Key questions

- Which digit do you divide first?
- How many groups of _____ hundreds/tens/ones are there in _____?
- If you cannot make a group in a column, what do you do?

Possible sentence stems

- _____ hundreds/tens/ones divided by _____ is equal to _____ hundreds/tens/ones with a remainder of _____

  The remainder is exchanged into _____ tens/ones.

Single age small step links

- Short division (Y5)
- Short division (Y6)

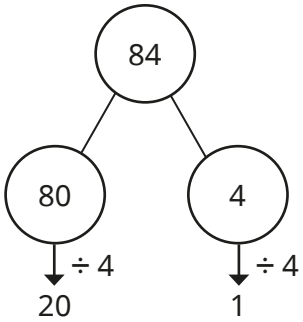
National Curriculum links

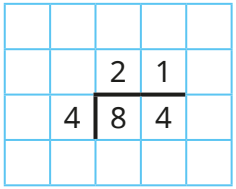
- Divide numbers up to four digits by a 1-digit number using the formal written method of short division and interpret remainders appropriately for the context (Y5)
- Divide numbers up to four digits by a 2-digit number using the formal written method of short division where appropriate, interpreting remainders according to the context (Y6)

Short division

Key learning

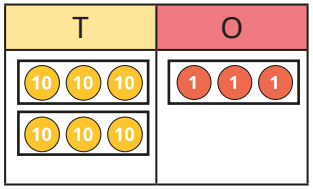
- What is the same and what is different about the two methods for dividing 84 by 4?

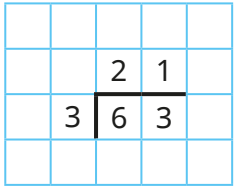




20 + 1 = 21, so 84 ÷ 4 = 21

- Complete the sentences to describe how the place value chart shows 63 ÷ 3





There are _____ groups of 3 tens.

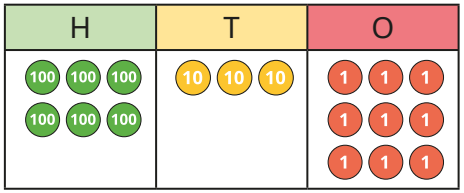
There is _____ group of 3 ones.

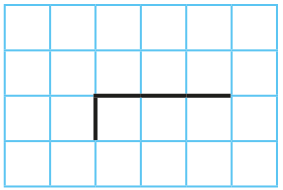
63 ÷ 3 = _____ + _____

= _____

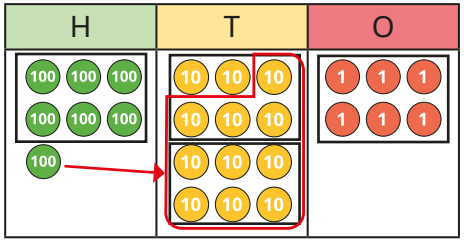
- Circle groups of 3 counters to work out 639 ÷ 3

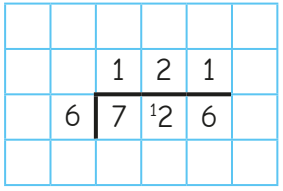
Complete the short division.





- Brett uses a place value chart and counters to work out 726 ÷ 6





Brett exchanges the remaining hundred counter for 10 ten counters.

Use Brett's method to work out the divisions.

729 ÷ 3 | 847 ÷ 7 | 615 ÷ 5

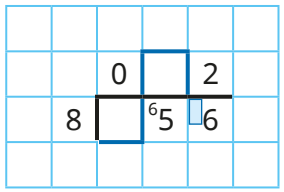
- Use short division to work out the calculations.

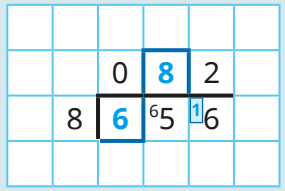
426 ÷ 3 | 946 ÷ 4 | 435 ÷ 5 | 656 ÷ 8

Short division

Reasoning and problem solving

Find the missing numbers.





Which calculation is the odd one out?

456 ÷ 4 | 465 ÷ 5

456 ÷ 6 | 456 ÷ 8

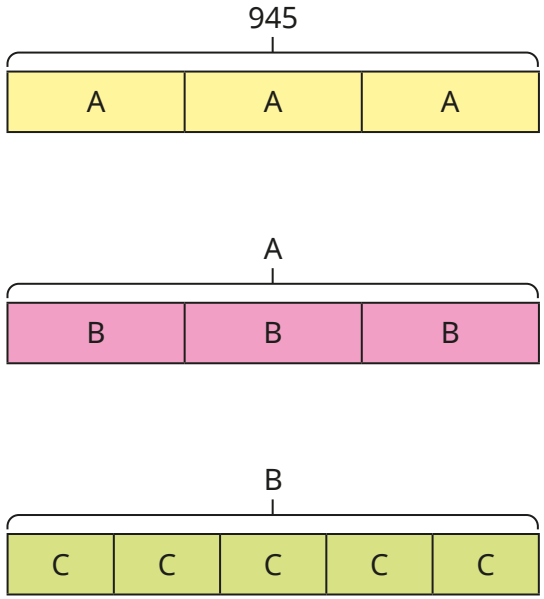
Find more than one answer.

Explain your answers.

multiple possible answers, e.g.

456 ÷ 4, as this only requires one exchange

Here are three bar models.
They are not drawn to scale.



Work out the value of C.

21

Divide a 4-digit number by a 1-digit number

Notes and guidance

In this small step, children progress to dividing a 4-digit number by a 1-digit number, including divisions with a remainder.

Place value counters continue to be used to represent calculations alongside the formal written method, so that children can visualise how one relates to the other. When there is a remainder, they can associate it with the amount "left over". The progression of divisions is carefully chosen to draw their attention to the link between the remainder and the number being divided by. They should generalise that a remainder must be less than the number being divided by. Remainders are represented in calculations as r1, r2 and so on.

Give Year 6 children the opportunity to interpret the remainder in context, for example knowing that "4 remainder 1" could mean 4 complete boxes with 1 left over, so 5 boxes are needed.

Things to look out for

- When dividing numbers that include zeros as placeholders, children may make errors with place value.
- Children may make the incorrect generalisation that the remainder is always 1

Key questions

- When do you need to make an exchange?
- What does the word "remainder" mean?

Possible sentence stems

- When dividing by _____, the greatest possible remainder is _____
- _____ ones divided by _____ ones = _____ ones remainder _____

Single age small step links

- Divide a 4-digit number by a 1-digit number (Y5)
- Divide with remainders (Y5)

- N/A

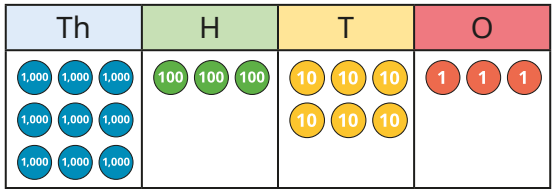
National Curriculum links

- Divide numbers up to four digits by a 1-digit number using the formal written method of short division and interpret remainders appropriately for the context (Y5)
- Divide numbers up to four digits by a 2-digit number using the formal written method of short division where appropriate, interpreting remainders according to the context (Y6)

Divide a 4-digit number by a 1-digit number

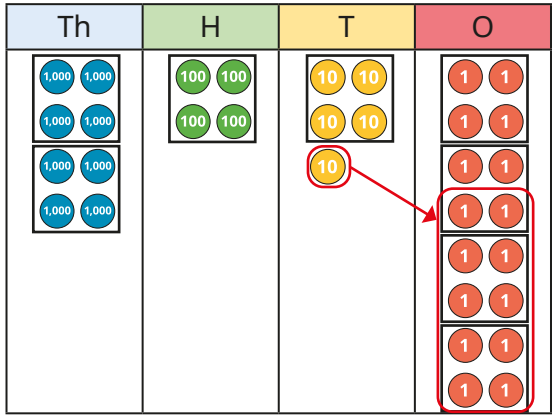
Key learning

- Use the place value chart to work out 9,363 ÷ 3



	3	9	3	6	3	

- Nijah has worked out 8,456 ÷ 4 using place value counters and short division.

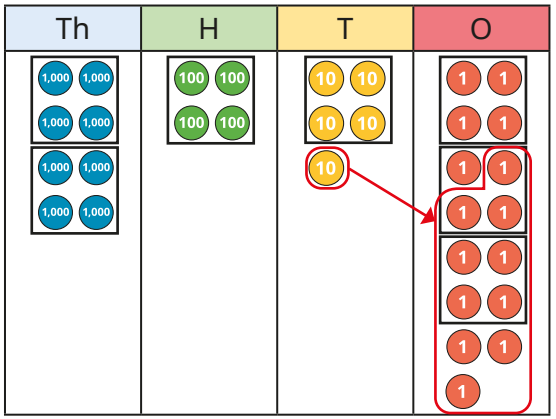


		2	1	1	4	
	4	8	4	5	$^{1}6$	

Use place value counters and short division to work out the divisions.

4,518 ÷ 3	8,406 ÷ 6	4,375 ÷ 7

- Mo is working out 8,455 ÷ 4



		2	1	1	3	r3
	4	8	4	5	$^{1}5$	

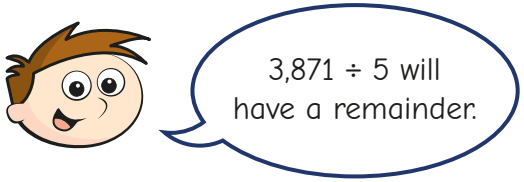
Use Mo's method to work out the divisions.

4,525 ÷ 3	8,406 ÷ 5	4,675 ÷ 9

- In a factory, muffins are packed into boxes of 4

One day, the factory makes 5,326 muffins.

How many muffins will **not** be boxed?

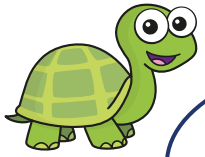
- 

Explain how Teddy knows this.

Divide a 4-digit number by a 1-digit number

Reasoning and problem solving

Tiny is working out 4,731 ÷ 8



This cannot be done because 8 is greater than each of the digits in the number.

Do you agree with Tiny?

Explain your answer.

No

Work out the divisions.

1,505 ÷ 5 1,506 ÷ 5 1,507 ÷ 5

1,508 ÷ 5 1,509 ÷ 5 1,510 ÷ 5

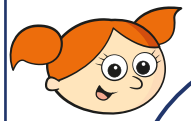
What do you notice about the remainders?

What does this tell you about remainders?

301, 301 r1, 301 r2

301 r3, 301 r4, 302

Alex is thinking of a 4-digit number less than 5,000



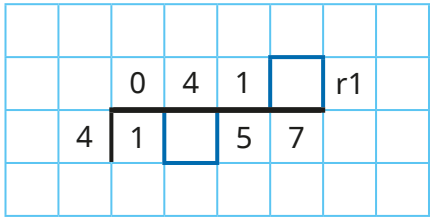
When my number is divided by 9, the remainder is 8. When my number is divided by 2, the remainder is 1. When my number is divided by 5, the remainder is 1

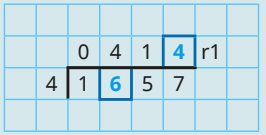
What could Alex's number be?

multiple possible answers, e.g.

4,571, 4,661, 4,751, 4,841, 4,931

Work out the missing digits.





Division using factors

Notes and guidance

In this small step, children build on their understanding of using factors in multiplication and learn to divide by a 2-digit number using repeated division.

Children start with the familiar strategy that, to divide by 4, they can halve and halve again. They then divide by multiples of 10, before looking at slightly more complex divisions using two 1-digit factors. Year 5 children may find it worthwhile to revise factor pairs, practising finding factor pairs of 2-digit numbers.

Establish that the divisions can be carried out in any order. This means that children can choose to divide first by the factor that they find easier to work with.

Things to look out for

- Children may partition the number they are dividing by into tens and ones instead of using factors.
- Children may factorise the number they are dividing by incorrectly.
- Children may identify 1 and the number itself as a pair of factors, which does not simplify the calculation.

Key questions

- What does the word "factor" mean?
- What are the factors of the number you are dividing by?
- How can you check your answer?
- Which factor are you going to divide by first/second? Why?

Possible sentence stems

- The factor pairs of _____ are _____
- To divide by _____, I can first divide by _____ and then divide the answer by _____

Single age small step links

- N/A

- Division using factors (Y6)

National Curriculum links

- Solve problems involving multiplication and division, including using their knowledge of factors and multiples, squares and cubes (Y5)
- Solve problems involving addition, subtraction, multiplication and division (Y6)

Division using factors

Key learning

- Take 24 counters and share them into two equal groups.

  Share each group into two equal groups.

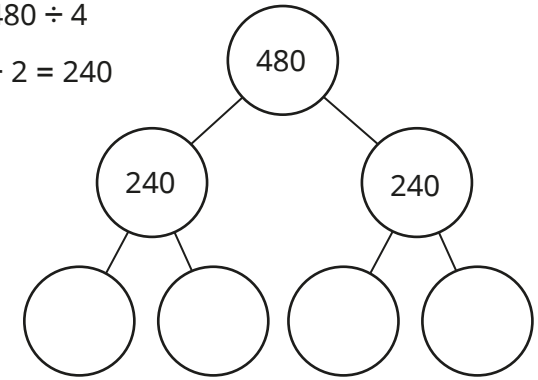
  How many groups do you have now?

  Complete the calculation.

  24 ÷ 2 ÷ 2 = 24 ÷ _____ = _____

- Filip is working out 480 ÷ 4

  He knows that 480 ÷ 2 = 240



  How can Filip use this fact to help find 480 ÷ 4?

- 100 counters are divided into 5 equal groups.

  How many counters are there in each group?

  The counters are then shared into 4 equal groups.

  How many counters are there in each group now?

- Complete the calculations.
  - 240 ÷ 30 = 240 ÷ 10 ÷ _____ = 24 ÷ _____ = _____
  - 240 ÷ 20 = 240 ÷ 10 ÷ _____ = 24 ÷ _____ = _____
  - 240 ÷ 40 = 240 ÷ 10 ÷ _____ = 24 ÷ _____ = _____

- Work out the divisions.

600 ÷ 30	720 ÷ 40	850 ÷ 50	760 ÷ 20

- Work out 570 ÷ 15 by dividing 570 by 5 and then dividing the result by 3

  Why does dividing a number by 5 and then dividing by 3 give you the same answer as dividing the number by 15?

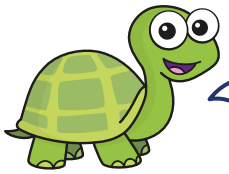
  Use this strategy to work out the divisions.

645 ÷ 15	770 ÷ 14	450 ÷ 18
875 ÷ 25	512 ÷ 16	1,104 ÷ 24

  Can any of the divisions be done in more than one way?

Division using factors

Reasoning and problem solving



To work out 1,325 ÷ 25, I will divide 1,325 by 5 and then divide the answer by 20

Explain why Tiny is wrong.

Tiny has partitioned 25 into 5 + 20 rather than using the factor pair 5 × 5 = 25

Dividing by 5 and then dividing by 20 is the same as dividing by 100

Use factor pairs to work out the divisions.

3,456 ÷ 48

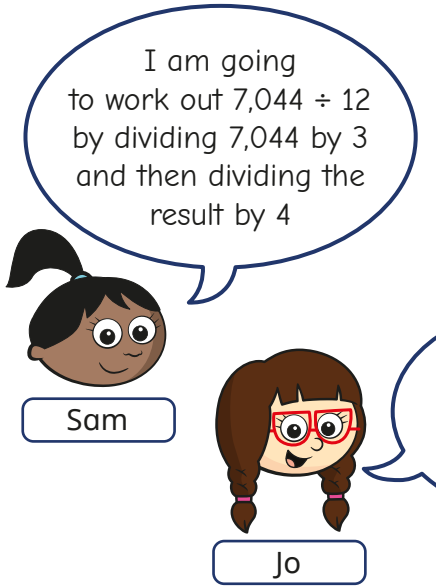
3,456 ÷ 24

3,456 ÷ 12

What do you notice about your answers?

72, 144, 288

When the number you are dividing by is halved, the answer is doubled.



Compare the children's methods.

Whose method do you prefer? Why?

Discuss as a class, recognising that each child gets the same answer.

Introduction to long division

Notes and guidance

In this small step, children are introduced to long division as a different method for dividing by a 2-digit number, including numbers that cannot be factorised into 1-digit numbers.

Year 5 children should spend more time focusing on dividing 3-digit numbers without remainders, using an expanded method that shows the multiples, before progressing to a more formal long division method.

Children divide 4-digit numbers, still without remainders, using their knowledge of multiplying by 10 and 100. Year 6 children could discuss which methods they prefer and find most efficient to complete different divisions. When dividing by composite numbers, it may be worth comparing the long division method with the method of division using factors covered in the previous step.

Long division with remainders is covered in the next step.

Things to look out for

- Children may need support in setting out the long divisions, for example by providing the questions on prepared squared grids with the questions already formatted.
- When dividing by prime numbers or large numbers, children may need support in working out the multiples of the number they are dividing by.

Key questions

- How can you use multiples to divide by a 2-digit number?
- Why do you subtract as you go along?
- What is the first step when performing a long division?

Possible sentence stems

- _____ thousands/hundreds/tens divided by _____ is equal to _____ thousands/hundreds/tens with a remainder of _____

  The remainder is exchanged into _____ hundreds/tens/ones.

Single age small step links

- N/A

- Introduction to long division (Y6)

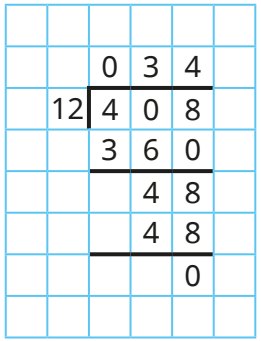
National Curriculum links

- Divide numbers up to four digits by a 2-digit whole number using the formal written method of long division, and interpret remainders as whole number remainders, fractions, or by rounding, as appropriate for the context (Y6)
- Solve problems involving addition, subtraction, multiplication and division (Y6)

Introduction to long division

Key learning

- Here is 408 ÷ 12 using the long division method.



(12 × 30)

(12 × 4)

Multiples of 12:

12 × 1 = 12

12 × 2 = 24

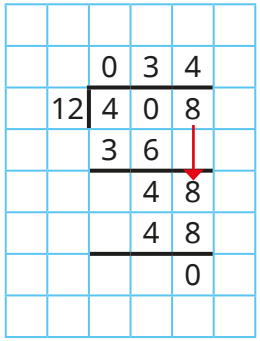
12 × 3 = 36

12 × 4 = 48

Use this method to work out the divisions.

600 ÷ 15 | 714 ÷ 17 | 684 ÷ 18

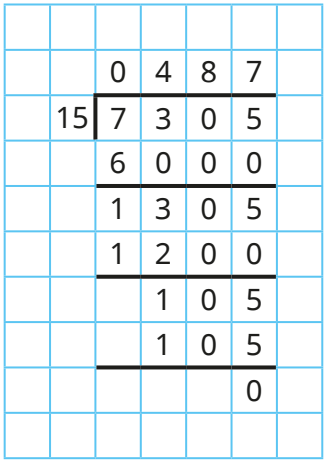
- Here is a different way of setting out a long division.



Use this method to work out the divisions.

858 ÷ 11 | 784 ÷ 14 | 627 ÷ 19

- Here is 7,305 ÷ 15 using the long division method.



(15 × 400)

(15 × 80)

(15 × 7)

Use this method to work out the divisions.

2,240 ÷ 16 | 1,845 ÷ 45 | 1,568 ÷ 16

- There are 816 players in a rugby tournament.

  Each team has 15 players and 2 substitute players.

  How many teams are playing in the tournament?

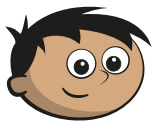
- 2,412 children are put into groups of 36

  How many groups of children are there?

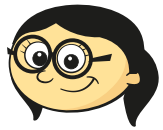
  Check your answer by dividing by factors.

Introduction to long division

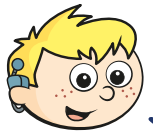
Reasoning and problem solving



Amir: I am going to work out 8,352 ÷ 12 by dividing 8,352 by 6 and then dividing the result by 2



Annie: I am going to work out 8,352 ÷ 12 using long division.



Max: I am going to work out 8,352 ÷ 12 using short division.

Compare the children's methods.

Whose method do you prefer? Why?

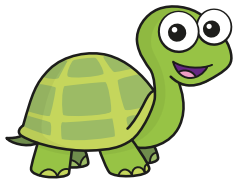
Discuss as a class, recognising that each child gets the same answer.

$7{,}560 \div 14 = 540$

Use the given calculation to work out the missing number.

$8{,}100 \div \_\_\_\_\_ = 540$

15



Tiny: 5,400 ÷ 24 is greater than 5,400 ÷ 25

Tiny is correct.

Work out how much greater 5,400 ÷ 24 is than 5,400 ÷ 25

9 greater

Long division with remainders

Notes and guidance

In this small step, children now move on to long divisions with remainders. It is important that they have a secure understanding of the long division method before starting this step.

This step includes context questions where children need to interpret the remainder and/or adjust the number they are dividing. For example, when thinking about packing items into boxes, they consider the number of full boxes or the total number of boxes needed.

Children should always check that the remainder is less than the number they are dividing by. They can use estimation as a sense-check for their answers. For example, 834 ÷ 18 is close to 800 ÷ 20, so the answer should be in the region of 40

Things to look out for

- Children may need support in setting out the long divisions, for example by providing the questions on prepared squared grids with the questions already formatted.
- When dividing by prime numbers or large numbers, children may need support in working out the multiples of the number they are dividing by.

Key questions

- Why do you subtract as you go along?
- In a long division, what happens after the subtractions if you cannot divide exactly?
- What is the first step when performing a long division?

Possible sentence stems

- _____ cannot be divided by _____, so there is a _____ of _____

Single age small step links

- N/A

- Long division with remainders (Y6)

National Curriculum links

- Divide numbers up to four digits by a 2-digit whole number using the formal written method of long division, and interpret remainders as whole number remainders, fractions, or by rounding, as appropriate for the context (Y6)
- Solve problems involving addition, subtraction, multiplication and division (Y6)

Long division with remainders

Key learning

- Dani uses multiples to help divide 364 by 15

		0	2	4	r	4
	15	3	6	4		
		3	0	0		
			6	4		
			6	0		
				4		

(15 × 20)

(15 × 4)

Multiples of 15:

15 × 1 = 15

15 × 2 = 30

15 × 3 = 45

15 × 4 = 60

Use Dani's method to work out the divisions.

206 ÷ 17 | 698 ÷ 21 | 900 ÷ 32

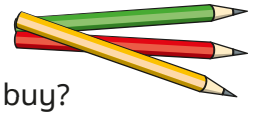
- Here is Ron's method for working out 1,429 ÷ 13

		0	1	0	9	r	12
	13	1	4	2	9		
		1	3	0			
			1	2	9		
			1	1	7		
				1	2		

Use this method to work out the divisions.

2,782 ÷ 16 | 6,723 ÷ 21 | 9,876 ÷ 23

- Mrs Smith needs to buy 490 pencils for her school.



Pencils are sold in boxes of 15

How many boxes of pencils does she need to buy?

Will Mrs Smith have any pencils spare?

How do you know?

- A factory produces 5,194 cups.

The cups are packed into boxes of 25 cups.

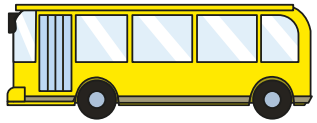
How many full boxes can be packed?

- 465 children and 28 adults need transport for a school trip.

A coach has seats for 48 people.

How many coaches are needed?

How many spare seats will there be?



- A portion of pasta is 75 g.

How many portions can be served from a 7 kg bag of pasta?

Will there be any pasta left over?

If yes, how much?

Long division with remainders

Reasoning and problem solving

Which calculations will definitely have a remainder?

7,431 ÷ 20

6,368 ÷ 15

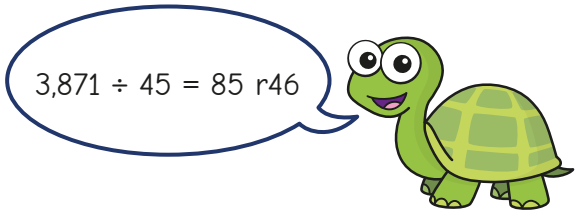
8,674 ÷ 18

9,315 ÷ 12

How do you know?

All the calculations will have a remainder.

Tiny is working out a division.



Explain why Tiny cannot be correct.

The remainder cannot be greater than 45

3,533 ÷ ☐ = 98 r5

What is the missing number?

Compare methods with a partner.

36

Solve problems with division

Notes and guidance

In this small step, children explore division problems, looking at the most appropriate strategy for finding a solution.

Children further develop their understanding of remainders when performing a division in context. For example, if pencils come in packs of 4 and a class needs 30 pencils, how many packs are needed? Children may recognise that they need to divide 30 by 4, which is equal to 7 r2. However, they then need to realise that this means that 8 packs are needed.

Children also look at alternative methods of division, such as partitioning the number into appropriate multiples of the number they are dividing by.

Encourage children to think about the numbers in a division question and to consider the most efficient method before they automatically start the formal method.

Things to look out for

- Children may try to use formal methods when alternative strategies would be more appropriate.

Key questions

- What is the most useful way of partitioning the number?
- Would you use short division or long division? Why?
- What are the factor pairs of _____? How does this help you to divide by _____? Which factor pair is easiest to use?

Possible sentence stems

- I will partition the number into _____ and _____, because both _____ and _____ are divisible by _____

Single age small step links

- Solve problems with multiplication and division (Y5)
- Solve problems with division (Y6)

National Curriculum links

- Solve problems involving multiplication and division, including using their knowledge of factors and multiples, squares and cubes (Y5)
- Solve problems involving addition, subtraction, multiplication and division (Y6)

Solve problems with division

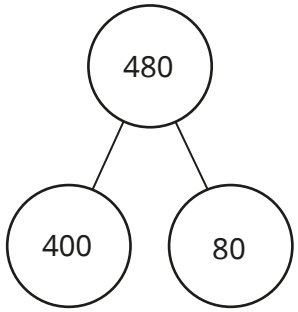
Key learning

- Complete the workings for 480 ÷ 4

  400 ÷ 4 = _____

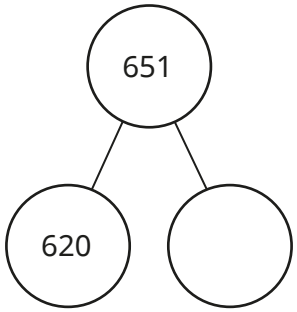
  80 ÷ 4 = _____

  So 480 ÷ 4 = _____ + _____ = _____

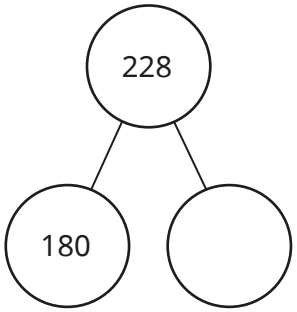


- Use partitioning to work out the divisions.

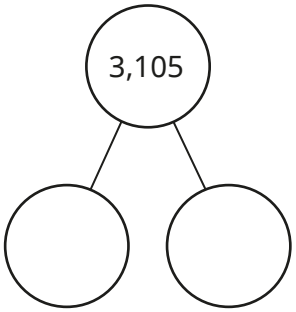
651 ÷ 31



228 ÷ 6



3,105 ÷ 15



- Which of the divisions can you work out mentally?

730 ÷ 10	842 ÷ 2	840 ÷ 20
6,792 ÷ 1	3,200 ÷ 800	399 ÷ 19

- Use your preferred method to work out the divisions.

560 ÷ 40	621 ÷ 3	8,000 ÷ 40
5,952 ÷ 24	9,624 ÷ 24	9,648 ÷ 24

Did you use the same method for each division?

- Esther has saved £9 in 20p coins.

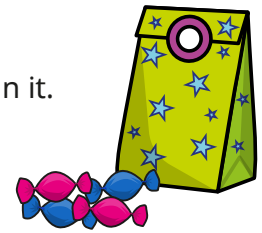
  How many 20p coins does she have?

- Tom is filling party bags.

  Each party bag will have exactly 12 sweets in it.

  Tom has 549 sweets altogether.

  How many party bags can he fill?



- Mr Xu needs 9,200 sheets of paper for the office printer.

  Paper is sold in packs of 900

  How many packs of paper does he need to buy?

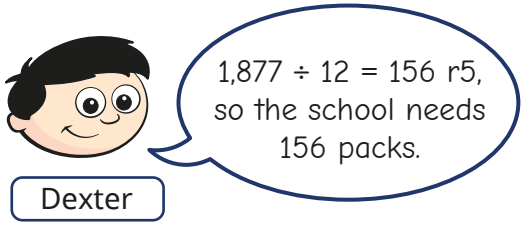
Solve problems with division

Reasoning and problem solving

Pencils come in packs of 12

A school needs 1,877 pencils.

Dexter and Eva are working out how many packs are needed.



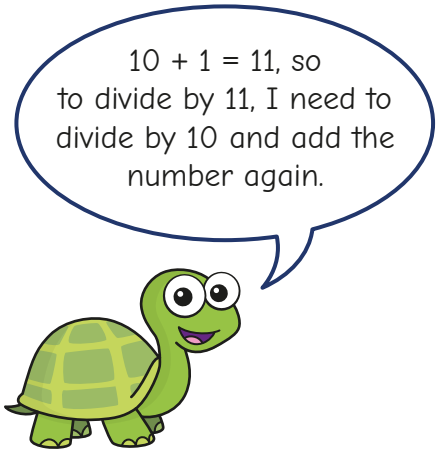
Who do you agree with?

Explain your answer.

Eva

The school will have to buy 157 packs, as 156 packs would not be enough, and it is not possible to buy r5 of a pack.

Tiny is trying to divide by 11



Explain why Tiny is wrong.

Tiny is confusing strategies for multiplication and division.

Efficient division

Notes and guidance

So far in this block, children have divided numbers in a range of contexts, using various methods. They have used informal methods to understand the structure of division and the formal written method to promote efficiency.

In this small step, children consolidate their knowledge and understanding of division, and make decisions regarding the most efficient or appropriate methods to use in a range of contexts.

They begin by looking at informal methods, such as partitioning, using known facts, factor pairs and number lines, and then compare these to the formal written method. Children make decisions about which method they prefer or which would be more efficient for a given problem.

Things to look out for

- Children may become over-reliant on the formal written method, instead of considering alternative approaches that may be more efficient.
- Children may partition the number being divided by, rather than using factors to break up the calculation, for example 12 ÷ 6 = 12 ÷ 4 ÷ 2 rather than 12 ÷ 6 = 12 ÷ 2 ÷ 3

Key questions

- Which method do you find the easiest/most efficient?
- How would you explain how this method works?
- What is the most efficient way to divide by _____?
- What happens if you double one factor and halve the other?
- How can you use factor pairs to help you?

Possible sentence stems

- To divide by _____, I can divide by _____ and then divide the result by _____

Single age small step links

- Efficient division (Y5)

- N/A

National Curriculum links

- Solve problems involving multiplication and division, including using their knowledge of factors and multiples, squares and cubes (Y5)
- Solve problems involving addition, subtraction, multiplication and division (Y6)

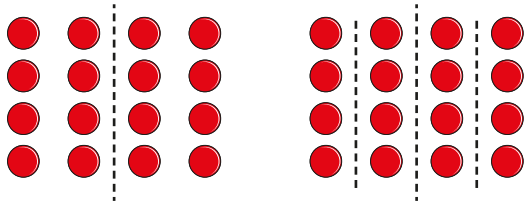
Efficient division

Key learning

- Complete the divisions.
  - 700 ÷ 1 = _____
  - 700 ÷ 10 = _____
  - 700 ÷ 100 = _____

  What do you notice?

- The array shows 12 ÷ 4 = 12 ÷ 2 ÷ 2



Make your own arrays to show the divisions.

24 ÷ 4 = 24 ÷ 2 ÷ 2	48 ÷ 8 = 48 ÷ 2 ÷ 2 ÷ 2

- Scott is using factors to work out 4,608 ÷ 6

Factors of 6 are 2 and 3:

4,608 ÷ 2 = 2,304 2,304 ÷ 3 = 768	4,608 ÷ 3 = 1,536 1,536 ÷ 2 = 768

So 4,608 ÷ 6 = 768

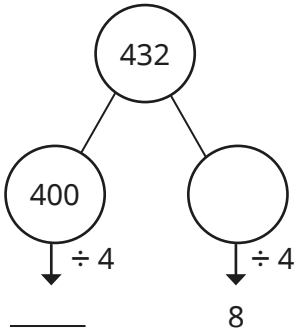
Use Scott's method to work out the divisions.

144 ÷ 6	948 ÷ 6	5,244 ÷ 6	5,922 ÷ 6

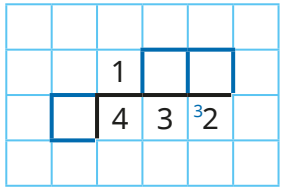
- Here are four different ways of working out 432 ÷ 4

Complete the calculations in each method.

Method 1: Partitioning



Method 2: Short division

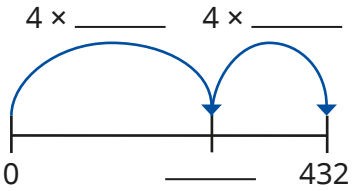


Method 3: Halve and halve again

432 ÷ _____ = 216

216 ÷ 2 = _____

Method 4: Finding groups of 4 along a number line



Which method would you use to work out these divisions?

220 ÷ 4	1,296 ÷ 6	2,002 ÷ 7	6,228 ÷ 18

Use your chosen method to work out each division.

Efficient division

Reasoning and problem solving

Is the statement true or false?

> All numbers that end in 2 are divisible by 2, all numbers that end in 5 are divisible by 5 and all numbers that end in zero are divisible by 10. So, all numbers that end in 4 will be divisible by 4

Explain your answer.

False

$24 \div 4 = 6$

What other division facts can you work out from this fact?

multiple possible answers, e.g.

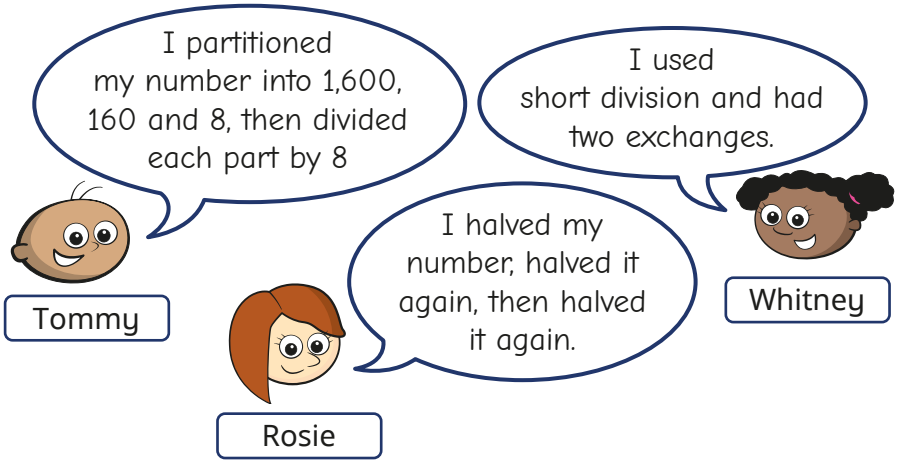
$240 \div 60 = 4$

$2{,}400{,}000 \div 4 = 600{,}000$

Tommy, Rosie and Whitney each choose one of the number cards.

2,448 | 280,000 | 1,768

They divide their number by 8



Which number card did each child choose?

Which method would you use to divide each number?

Tommy: 1,768 Rosie: 280,000 Whitney: 2,448

Solve multi-step problems

Notes and guidance

In this small step, children apply the skills they have developed so far in this block to solving problems in real-life contexts.

The problems involve more than one calculation and children must decide which operations they need to perform and in what order. This will need careful modelling.

Year 5 children may benefit from practising the formal methods for multiplication and division first, before applying these to solve multi-step problems. Encourage children to think about the best way to perform any of the calculations and to use the most appropriate written, informal or mental method. For example, this might include using a number line to work out a subtraction after a long multiplication.

Things to look out for

- In longer problems, children may find the number of words overwhelming and need encouragement to split the problem into smaller parts.
- Children may find choosing the correct operation difficult.
- Children may be able to divide using a procedure, but lack understanding of the remainder in a particular context.

Key questions

- What can you work out first?
- How can you draw a diagram to represent the problem?
- Can you work out the answer to this part of the problem mentally or do you need another method?

Possible sentence stems

- First/next I need to work out _____

  The calculation I need to do is _____

Single age small step links

- Solve problems with multiplication and division (Y5)
- Solve multi-step problems (Y6)

National Curriculum links

- Solve problems involving addition, subtraction, multiplication and division and a combination of these, including understanding the meaning of the equals sign (Y5)
- Solve problems involving addition, subtraction, multiplication and division (Y6)

Solve multi-step problems

Key learning

- A football pitch is 91 m long and 47 m wide.

  What is the area of the pitch?

  The area of a field is 30,000 m^2

  How many football pitches could fit on the field?

  How do you know what calculation to do?

- A train has 13 carriages.

  Each carriage can carry 57 people.

  639 people have reserved a seat.

  How many unreserved seats are there?

- The total mass of books in a box is 35 kg.

  The total mass of magazines in a box is 34 kg.

  - There are 45 boxes of books and 36 boxes of magazines in a warehouse.

    What is the total mass of books and magazines?

  - A shop receives 315 kg of books and 714 kg of magazines.

    How many boxes does the shop receive?

- A coach has 48 seats and a minibus has 17 seats.

  435 people from a school go on a trip.

  A teacher books 7 coaches and 8 minibuses.

  How many spare seats will there be?

- Huan can type 35 words a minute.

  How many words can he type in an hour?

  How long does it take Huan to type 1,000 words?

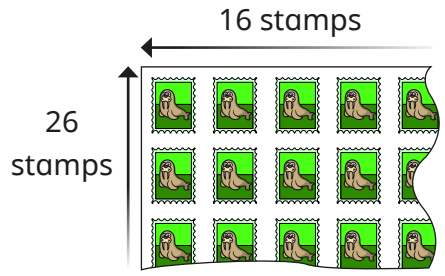
- A headteacher has £3,000 to spend on new furniture.

  She wants to buy 15 desks for £98 each and 34 chairs for £56 each.

  Does she have enough money?

- A sheet of stamps has 26 rows and 16 columns of stamps.

  How many stamps are there altogether on 42 sheets?



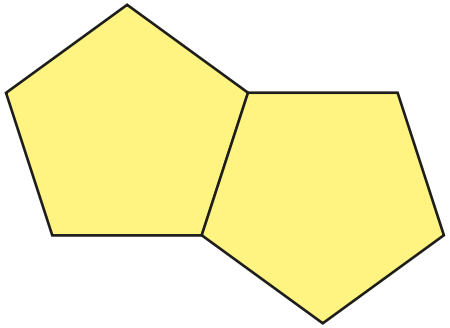
Solve multi-step problems

Reasoning and problem solving

Aisha makes a shape using two regular pentagons.

The perimeter of the shape is 5,232 mm.

What is the perimeter of one of the pentagons?



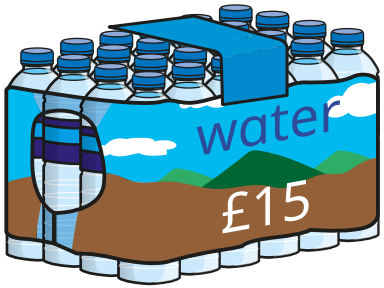
Aisha adds another regular pentagon to the shape.

What is the perimeter now?

3,270 mm

7,848 mm

24 bottles of water cost £15



How many water bottles can you buy for £45?

How many water bottles can you buy for £450?

How many water bottles can you buy for £960?

How much will 888 bottles of water cost?

72

720

1,536

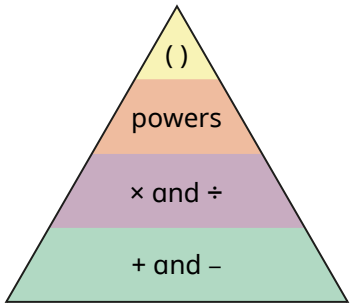
£555

Order of operations

Notes and guidance

In this small step, children learn the order of priority for operations in a calculation: that calculations in brackets should always be done first, and that multiplication and division have equal priority and should be performed before additions and subtractions.

This image may be useful when teaching the order of operations.



Things to look out for

- If children have heard acronyms such as BIDMAS or BODMAS, they may mistakenly think that addition should be done before subtraction and incorrectly work out, for example, 10 – 3 + 4 as 10 – 7 = 3
  Similarly, children may not be aware that multiplication and division are of equal priority.

Key questions

- Does it make a difference if you perform the operations in a different order?
- What do brackets in a calculation mean? What would happen if you did not use the brackets?
- Which operation has greater priority, _____ or _____?
- How many pairs of operations have equal priority?

Possible sentence stems

- _____ has greater priority than _____, so the first part of the calculation I need to do is _____

Single age small step links

- N/A

- Order of operations (Y6)

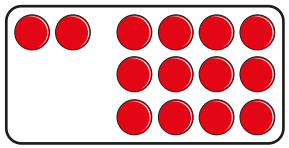
National Curriculum links

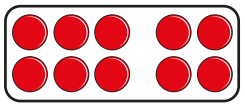
- Perform mental calculations, including with mixed operations and large numbers (Y6)
- Use their knowledge of the order of operations to carry out calculations involving the four operations (Y6)

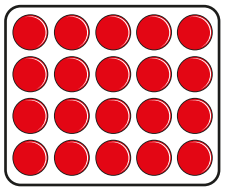
Order of operations

Key learning

- Match the counters to the calculations.







$2 + 3 \times 4$ $2 \times 3 + 4$ $(2 + 3) \times 4$

- Draw counters to represent each calculation.

$5 + 2 \times 3$ $(5 + 2) \times 3$

Work out the answers.

- Work out the calculations.

$(5 + 4) \times 6$ $10 + 4 \div 2$ $12 - 4 \div 2$

$5 + 4 \times 6$ $(10 + 4) \div 2$ $12 - 4 \div 2$

- Add brackets to make the calculations correct.
  - $5 + 4 \times 6 = 54$
  - $15 \times 4 - 3 = 15$
  - $50 - 20 \times 2 = 60$
  - $90 \div 9 + 1 = 9$

- Work out the calculations.

$7 \times 3 + 4 \times 2$ $7 \times 3 - 4 \times 2$ $7 \times (3 + 4) \times 2$

- Brett has 6 bags with 8 sweets in each bag.

He adds one more sweet to each bag.

Which calculation shows how many sweets there are in total?

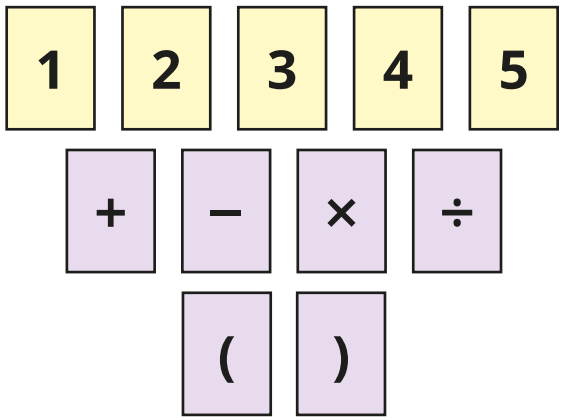
$6 \times (8 + 1)$ $6 \times 8 + 1$

- Work out the calculations.

$5^2 + 6 \times 5$ $5^2 \div (6 - 5)$ $(6 + 5)^2$

Order of operations

Reasoning and problem solving



Use some of the digits and symbols to write as many calculations as you can that give different answers.

Is it possible to make every number from zero to 20?

multiple possible answers, e.g.

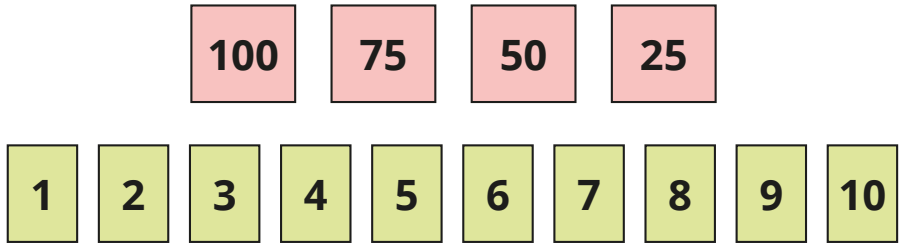
$1 \times 2 \times 3 + 4 = 10$

$(1 + 2) \times 3 + 4 = 13$

$(1 + 2) \times (3 + 5) = 24$

$(1 + 2 + 3) \times 5 = 30$

Here are some number cards.



Pick **one** large number from the top row.

Pick **five** smaller numbers from the bottom row.

Use a calculator or computer to generate a 3-digit target number.

Use your numbers, the four operations and brackets to find a number as close as possible to the target number.

Compare answers as a class.

Mental calculations and estimation

Notes and guidance

Children should use mental strategies and estimation whenever appropriate, and several examples have been included throughout the block.

This small step reminds them of the importance of mental strategies and estimation, and gives them an opportunity to revisit and extend their learning from this block and previous years.

Children should be aware that estimating the answer to a calculation serves as a sense-check on whether their answer is correct, and that this can be done either before or after a calculation. The numbers they choose when performing estimates should be simple enough for this to be done mentally.

Make links back to previous learning on rounding when simplifying numbers within a calculation.

Things to look out for

- Children may try to use formal methods when alternative strategies would be more appropriate.
- Children may not round numbers to an appropriate degree of accuracy.

Key questions

- Should you round the number to the nearest 10/100/1,000? Why?
- What number is (for example) 99 close to? How does this help with the calculation? What adjustment do you need to make?
- Why are estimates to the answers of calculations useful?

Possible sentence stems

- The previous/next multiple of _____ is _____

  _____ rounded to the nearest _____ is _____

Single age small step links

- N/A

- Mental calculations and estimation (Y6)

National Curriculum links

- Use rounding to check answers to calculations and determine, in the context of a problem, levels of accuracy (Y5)
- Use estimation to check answers to calculations and determine, in the context of a problem, an appropriate degree of accuracy (Y6)

Mental calculations and estimation

Key learning

- Complete the table.

Number	Rounded to the nearest 10	Rounded to the nearest 100	Rounded to the nearest 1,000
5,979			
1,089			
5,424			

- Use rounding to estimate the answer to each calculation.

899 × 48 | 5,967 × 42 | 8,134 ÷ 11

Compare answers with a partner.

- What strategies would you use to find the exact answers to the calculations?

360 × 20 | 360 ÷ 20 | 899 × 48

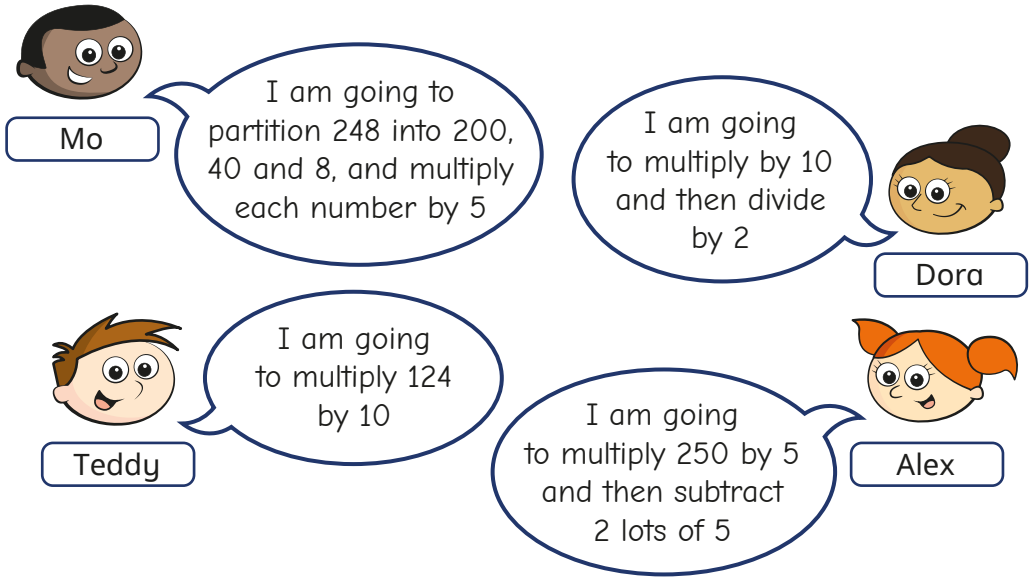
Compare answers with a partner.

- How could you change the order of the numbers in each calculation to make them easier to complete mentally?

764 × 2 × 5 | 25 × 18 × 4 | 2 × 67 × 50

Work out the answers to the calculations.

- The children are using different methods to work out 248 × 5



Use each method to work out 498 × 5

Whose method do you prefer? Why?

Mental calculations and estimation

Reasoning and problem solving

Here is a number line.

Estimate the number shown by arrow B for these values of A and C:

- A = 0 and C = 1,000
- A = 60 and C = 260
- A = 2 and C = 3
- A = 1,000 and C = 100,000

B is approximately one-tenth of the way from A to C, so answers should be around:

100

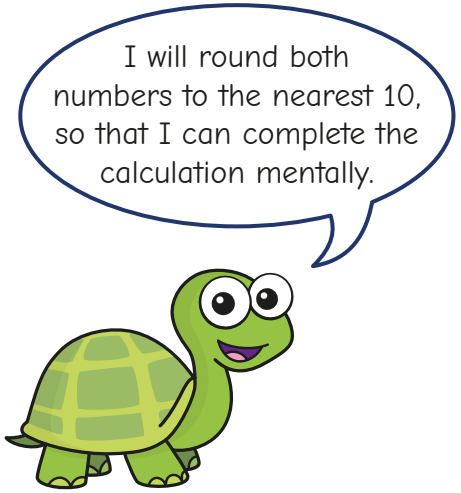
80

2.1

10,000

Tiny is using rounding to estimate the answer to the calculation.

4,965 × 72



Do you think that Tiny's method is the most efficient?

Explain your answer.

No

Reason from known facts

Notes and guidance

In this small step, children work out other facts from a given fact, using their knowledge of place value, inverse operations, commutativity and the mental strategies practised in this block, particularly in the previous small step.

Year 5 children should use bar models to help them to see the links between the different calculations and allow for flexible thinking to work out other related facts.

Children need to be confident in multiplying and dividing by powers of 10. They also explore the idea of doubling and halving.

It is important that Year 6 children can not only work out the answer of a related fact, but also explain the connections between calculations that helped them to arrive at this answer.

Things to look out for

- Children may try to calculate the answers instead of looking at the relationships between the calculations and using reasoning.
- Children may over-generalise and try to use multiplication strategies that do not work for other operations.
- Children may need support to see the connections between a given fact and the adjusted calculation.

Key questions

- How can you use an inverse operation to find related facts?
- What is the same and what is different about the numbers in the given calculation and the numbers in the calculation you want to work out?
- How will the answer change if you increase/decrease/multiply/divide one/both of the numbers by _____?

Possible sentence stems

- If I multiply/divide one of the numbers in the calculation by _____, then the answer will change by _____

Single age small step links

- N/A

- Reason from known facts (Y6)

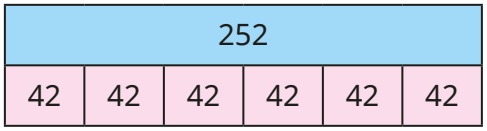
National Curriculum links

- Multiply and divide mentally, drawing upon known facts (Y5)
- Perform mental calculations, including with mixed operations and large numbers (Y6)

Reason from known facts

Key learning

- Write four facts shown by the bar model.



- Use the fact that 4 × 4 = 16 to work out the answers to the calculations.

4 × 8 | 4 × 16 | 4 × 32 | 4 × 64

4 × 128 | 4 × 256 | 8 × 256

- Use the fact that 13 × 21 = 273 to work out the answers to the calculations.

13 × 210 | 130 × 21 | 130 × 210

273 ÷ 21 | 273 ÷ 13

- Use the fact that 8,100 ÷ 25 = 324 to work out the answers to the calculations.

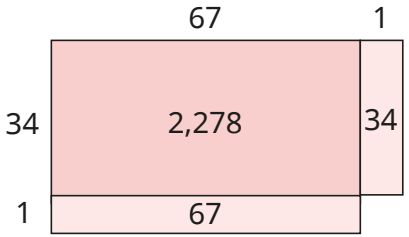
8,100 ÷ 324 | 324 × 250 | 3,240 × 250

81,000 ÷ 324 | 8,100 ÷ 250

- Use the fact that 34 × 67 = 2,278 to work out the multiplications.

67 × 35 | 68 × 35

You can use the area model to help you.

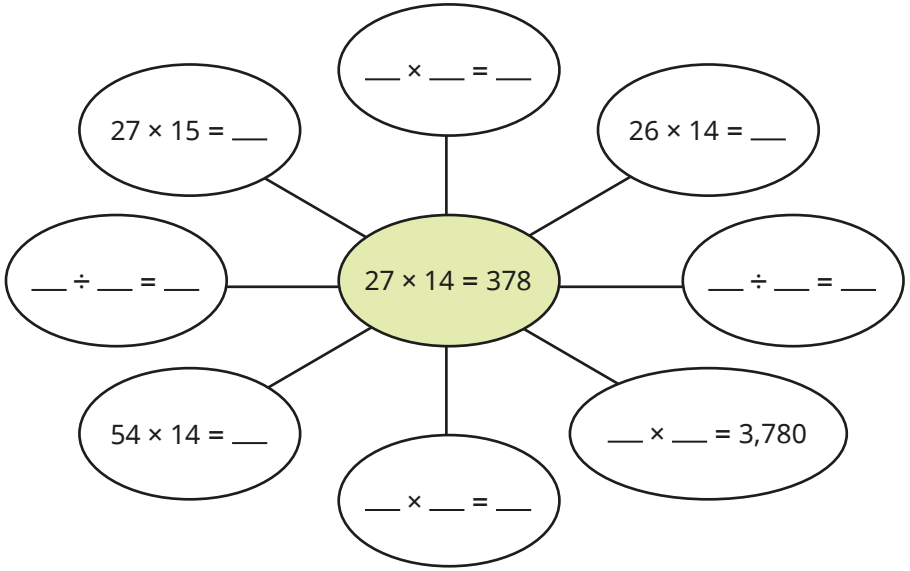


- Use the fact that 4,144 ÷ 16 = 259 to work out 17 × 259

Reason from known facts

Reasoning and problem solving

Complete the spider diagram.



Compare methods with a partner.

27 × 15 = **405**

26 × 14 = **364**

54 × 14 = **756**

270 × 14 = **3,780** or 27 × 140 = **3,780**

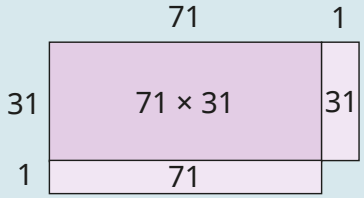
Without working them out, which calculation has the greater answer?

72 × 31

71 × 32

Draw a diagram to explain how you know.

Compare both calculations to 71 × 31



71 × 32 is 71 greater than 71 × 31

72 × 31 is 31 greater than 71 × 31

So 71 × 32 is greater than 72 × 31