



Shottery St Andrew's C of E Primary School

Calculation policy, UPPER KS2

The following pages show the progression in calculation (addition, subtraction, multiplication and division) and how this works in line with the National Curriculum. The consistent use of the CPA (concrete, pictorial, abstract) helps children develop mastery across all the operations in an efficient and reliable way. This policy shows how these methods develop children's confidence in their understanding of both written and mental methods.

KEY STAGE 2

In upper Key Stage 2, children build on secure foundations in calculation, and develop fluency, accuracy and flexibility in their approach to the four operations. They work with whole numbers and adapt their skills to work with decimals, and they continue to develop their ability to select appropriate, accurate and efficient operations.

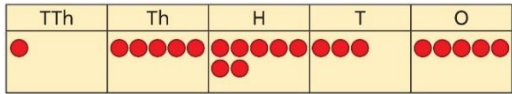
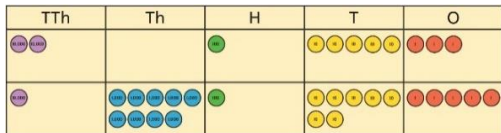
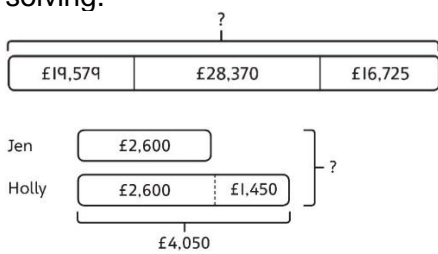
Key language: decimal, column methods, exchange, partition, mental method, ten thousand, hundred thousand, million, factor, multiple, prime number, square number, cube number


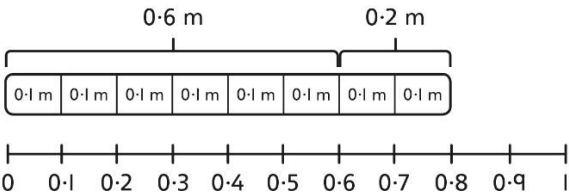
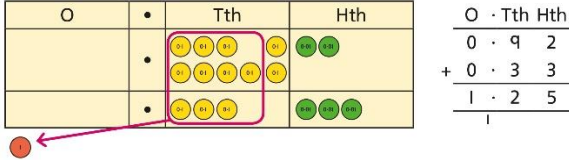
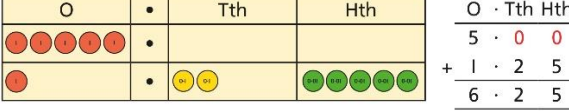
Addition and subtraction: Children build on their column methods to add and subtract numbers with up to seven digits, and they adapt the methods to calculate efficiently and effectively with decimals, ensuring understanding of place value at every stage. Children compare and contrast methods, and they select mental methods or jottings where appropriate and where these are more likely to be efficient or accurate when compared with formal column methods. Bar models are used to represent the calculations required to solve problems and may indicate where efficient methods can be chosen.

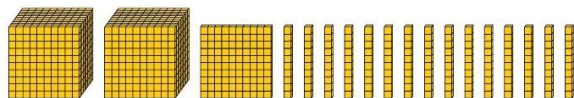
Multiplication and division: Building on their understanding, children develop methods to multiply up to 4-digit numbers by single-digit and 2-digit numbers. Children develop column methods with an understanding of place value, and they continue to use the key skill of unitising to multiply and divide by 10, 100 and 1,000. Written division methods are introduced and adapted for division by single-digit and 2-digit numbers and are understood alongside the area model and place value. In Year 6, children develop a secure understanding of how division is related to fractions. Multiplication and division of decimals are also introduced and refined in Year 6.

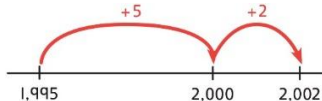


























































Fractions: Children find fractions of amounts, multiply a fraction by a whole number and by another fraction, divide a fraction by a whole number, and add and subtract fractions with different denominators. Children become more confident working with improper fractions and mixed numbers and can calculate with them. Understanding of decimals with up to 3 decimal places is built through place value and as fractions, and children calculate with decimals in the context of measure as well as in pure arithmetic. Children develop an understanding of percentages in relation to hundredths, and they understand how to work with common percentages: 50%, 25%, 10% and 1%.

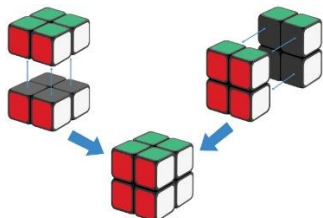
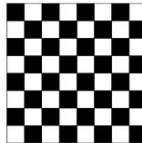
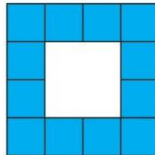


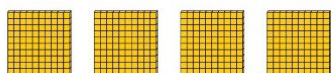


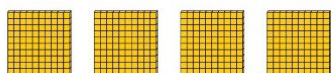
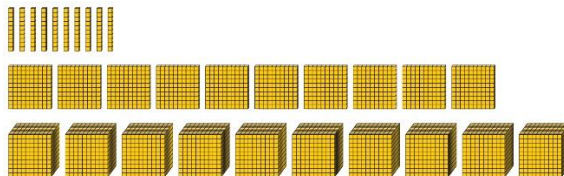


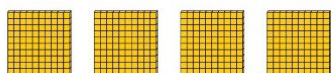
Year 5

	Concrete	Pictorial	Abstract
Year 5 Addition			
Column addition with whole numbers	<p>Use place value equipment to represent additions.</p> <p><i>Add a row of counters onto the place value grid to show $15,735 + 4,012$.</i></p> 	<p>Represent additions, using place value equipment on a place value grid alongside written methods.</p>  <p><i>I need to exchange 10 tens for a 100.</i></p> $\begin{array}{r} \text{TTh} \text{ Th} \text{ H} \text{ T} \text{ O} \\ 2 \quad 0 \quad 1 \quad 5 \quad 3 \\ + 1 \quad 9 \quad 1 \quad 7 \quad 5 \\ \hline 3 \quad 9 \quad 3 \quad 2 \quad 8 \end{array}$	<p>Use column addition, including exchanges.</p> $\begin{array}{r} \text{TTh} \text{ Th} \text{ H} \text{ T} \text{ O} \\ 1 \quad 9 \quad 1 \quad 7 \quad 5 \\ + 1 \quad 8 \quad 4 \quad 1 \quad 7 \\ \hline 3 \quad 7 \quad 5 \quad 9 \quad 2 \end{array}$
Representing additions		<p>Bar models represent addition of two or more numbers in the context of problem solving.</p>  $\begin{array}{r} \text{Th} \text{ H} \text{ T} \text{ O} \\ 2 \quad 6 \quad 0 \quad 0 \\ + 1 \quad 4 \quad 5 \quad 0 \\ \hline 4 \quad 0 \quad 5 \quad 0 \end{array}$ $\begin{array}{r} \text{Th} \text{ H} \text{ T} \text{ O} \\ 2 \quad 6 \quad 0 \quad 0 \\ + 4 \quad 0 \quad 5 \quad 0 \\ \hline 6 \quad 6 \quad 5 \quad 0 \end{array}$	<p>Use approximation to check whether answers are reasonable.</p> $\begin{array}{r} \text{TTh} \text{ Th} \text{ H} \text{ T} \text{ O} \\ 2 \quad 3 \quad 4 \quad 0 \quad 5 \\ + \quad 7 \quad 8 \quad 9 \quad 2 \\ \hline 2 \quad 0 \quad 2 \quad 9 \quad 7 \end{array}$ $\begin{array}{r} \text{TTh} \text{ Th} \text{ H} \text{ T} \text{ O} \\ 2 \quad 3 \quad 4 \quad 0 \quad 5 \\ + \quad 7 \quad 8 \quad 9 \quad 2 \\ \hline 3 \quad 1 \quad 2 \quad 9 \quad 7 \end{array}$ <p><i>I will use $23,000 + 8,000$ to check.</i></p>

<p>Adding tenths</p>	<p>Link measure with addition of decimals.</p> <p>Two lengths of fencing are 0.6 m and 0.2 m.</p> <p>How long are they when added together?</p> 	<p>Use a bar model with a number line to add tenths.</p>  <p>$0.6 + 0.2 = 0.8$ 6 tenths + 2 tenths = 8 tenths</p>	<p>Understand the link with adding fractions.</p> $\frac{6}{10} + \frac{2}{10} = \frac{8}{10}$ <p>6 tenths + 2 tenths = 8 tenths $0.6 + 0.2 = 0.8$</p>
<p>Adding decimals using column addition</p>	<p>Use place value equipment to represent additions.</p> <p>Show $0.23 + 0.45$ using place value counters.</p>	<p>Use place value equipment on a place value grid to represent additions.</p> <p>Represent exchange where necessary.</p>  <p>Include examples where the numbers of decimal places are different.</p> 	<p>Add using a column method, ensuring that children understand the link with place value.</p> $\begin{array}{r} \text{O} \cdot \text{Tth} \text{Hth} \\ 0 \cdot 2 \quad 3 \\ + 0 \cdot 4 \quad 5 \\ \hline 0 \cdot 6 \quad 8 \end{array}$ <p>Include exchange where required, alongside an understanding of place value.</p> $\begin{array}{r} \text{O} \cdot \text{Tth} \text{Hth} \\ 0 \cdot 9 \quad 2 \\ + 0 \cdot 3 \quad 3 \\ \hline 1 \cdot 2 \quad 5 \end{array}$ <p>Include additions where the numbers of decimal places are different.</p> <p>$3.4 + 0.65 = ?$</p> $\begin{array}{r} \text{O} \cdot \text{Tth} \text{Hth} \\ 3 \cdot 4 \quad 0 \\ + 0 \cdot 6 \quad 5 \\ \hline \end{array}$

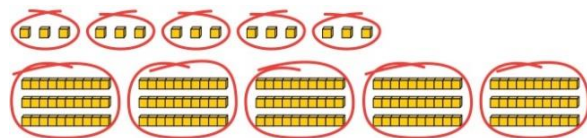
Year 5 Subtraction																																																																																																																																																																													
Column subtraction with whole numbers	<p>Use place value equipment to understand where exchanges are required.</p> <p>2,250 – 1,070</p> 	<p>Represent the stages of the calculation using place value equipment on a grid alongside the calculation, including exchanges where required.</p> <p>15,735 – 2,582 = 13,153</p> <table border="1" data-bbox="956 494 1368 563"><thead><tr><th>TTh</th><th>Th</th><th>H</th><th>T</th><th>O</th></tr></thead><tbody><tr><td>1</td><td>5</td><td>7</td><td>3</td><td>5</td></tr><tr><td></td><td></td><td></td><td></td><td></td></tr><tr><td></td><td></td><td></td><td></td><td></td></tr><tr><td></td><td></td><td></td><td></td><td></td></tr><tr><td></td><td></td><td></td><td></td><td></td></tr></tbody></table> <p>Now subtract the 10s. Exchange 1 hundred for 10 tens.</p> <table border="1" data-bbox="956 632 1368 721"><thead><tr><th>TTh</th><th>Th</th><th>H</th><th>T</th><th>O</th></tr></thead><tbody><tr><td>1</td><td>5</td><td>7</td><td>3</td><td>5</td></tr><tr><td></td><td></td><td></td><td></td><td></td></tr><tr><td></td><td></td><td></td><td></td><td></td></tr><tr><td></td><td></td><td></td><td></td><td></td></tr><tr><td></td><td></td><td></td><td></td><td></td></tr></tbody></table> <p>Subtract the 100s, 1,000s and 10,000s.</p> <table border="1" data-bbox="956 770 1368 860"><thead><tr><th>TTh</th><th>Th</th><th>H</th><th>T</th><th>O</th></tr></thead><tbody><tr><td>1</td><td>5</td><td>7</td><td>3</td><td>5</td></tr><tr><td></td><td></td><td></td><td></td><td></td></tr><tr><td></td><td></td><td></td><td></td><td></td></tr><tr><td></td><td></td><td></td><td></td><td></td></tr><tr><td></td><td></td><td></td><td></td><td></td></tr></tbody></table> <table border="1" data-bbox="1384 494 1532 592"><thead><tr><th>TTh</th><th>Th</th><th>H</th><th>T</th><th>O</th></tr></thead><tbody><tr><td>1</td><td>5</td><td>7</td><td>3</td><td>5</td></tr><tr><td>–</td><td>2</td><td>5</td><td>8</td><td>2</td></tr><tr><td></td><td></td><td></td><td></td><td>3</td></tr></tbody></table> <table border="1" data-bbox="1384 632 1532 734"><thead><tr><th>TTh</th><th>Th</th><th>H</th><th>T</th><th>O</th></tr></thead><tbody><tr><td>1</td><td>5</td><td>7</td><td>3</td><td>5</td></tr><tr><td>–</td><td>2</td><td>5</td><td>8</td><td>2</td></tr><tr><td></td><td></td><td></td><td></td><td>3</td></tr></tbody></table> <table border="1" data-bbox="1384 761 1532 863"><thead><tr><th>TTh</th><th>Th</th><th>H</th><th>T</th><th>O</th></tr></thead><tbody><tr><td>1</td><td>5</td><td>7</td><td>3</td><td>5</td></tr><tr><td>–</td><td>2</td><td>5</td><td>8</td><td>2</td></tr><tr><td></td><td>1</td><td>3</td><td>1</td><td>5</td></tr></tbody></table>	TTh	Th	H	T	O	1	5	7	3	5																					TTh	Th	H	T	O	1	5	7	3	5																					TTh	Th	H	T	O	1	5	7	3	5																					TTh	Th	H	T	O	1	5	7	3	5	–	2	5	8	2					3	TTh	Th	H	T	O	1	5	7	3	5	–	2	5	8	2					3	TTh	Th	H	T	O	1	5	7	3	5	–	2	5	8	2		1	3	1	5	<p>Use column subtraction methods with exchange where required.</p> <table border="1" data-bbox="1554 357 1771 496"><thead><tr><th>TTh</th><th>Th</th><th>H</th><th>T</th><th>O</th></tr></thead><tbody><tr><td>5</td><td>8</td><td>1</td><td>0</td><td>9</td></tr><tr><td>–</td><td>1</td><td>8</td><td>5</td><td>3</td></tr><tr><td></td><td>4</td><td>3</td><td>5</td><td>6</td></tr></tbody></table> <p>62,097 – 18,534 = 43,563</p>	TTh	Th	H	T	O	5	8	1	0	9	–	1	8	5	3		4	3	5	6
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Checking strategies and representing subtractions		<p>Bar models represent subtractions in problem contexts, including ‘find the difference’.</p> <p>Athletics Stadium 75,450</p> <p>Hockey Centre 42,300</p> <p>Velodrome 15,735 ?</p>	<p>Children can explain the mistake made when the columns have not been ordered correctly.</p> <table border="1" data-bbox="1554 1018 1711 1155"><caption>Bella's working</caption><thead><tr><th>TTh</th><th>Th</th><th>H</th><th>T</th><th>O</th></tr></thead><tbody><tr><td>1</td><td>7</td><td>8</td><td>7</td><td>7</td></tr><tr><td>+</td><td>4</td><td>0</td><td>1</td><td>2</td></tr><tr><td></td><td>5</td><td>7</td><td>9</td><td>9</td></tr></tbody></table> <table border="1" data-bbox="1742 1018 1890 1155"><caption>Correct method</caption><thead><tr><th>TTh</th><th>Th</th><th>H</th><th>T</th><th>O</th></tr></thead><tbody><tr><td>1</td><td>7</td><td>8</td><td>7</td><td>7</td></tr><tr><td>+</td><td></td><td>4</td><td>0</td><td>1</td></tr><tr><td></td><td>2</td><td>1</td><td>8</td><td>8</td></tr></tbody></table> <p>Use approximation to check calculations.</p> <p>I calculated 18,000 + 4,000 mentally to check my subtraction.</p>	TTh	Th	H	T	O	1	7	8	7	7	+	4	0	1	2		5	7	9	9	TTh	Th	H	T	O	1	7	8	7	7	+		4	0	1		2	1	8	8																																																																																																																																		
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Choosing efficient methods			<p>To subtract two large numbers that are close, children find the difference by counting on.</p> <p>$2,002 - 1,995 = ?$</p> <div></div> <p>Use addition to check subtractions.</p> <p><i>I calculated $7,546 - 2,355 = 5,191$.</i></p> <p><i>I will check using the inverse.</i></p>																																																									
Subtracting decimals	<p>Explore complements to a whole number by working in the context of length.</p> <div></div> <p>1 m - <input type="text"/> m = <input type="text"/> m</p> <p>$1 - 0.49 = ?$</p>	<p>Use a place value grid to represent the stages of column subtraction, including exchanges where required.</p> <p>$5.74 - 2.25 = ?$</p> <div><table><tr><th>O</th><th>•</th><th>Tth</th><th>Hth</th></tr><tr><td></td><td>•</td><td> </td><td></td></tr></table><p>Exchange 1 tenth for 10 hundredths.</p><table><tr><th>O</th><th>•</th><th>Tth</th><th>Hth</th></tr><tr><td></td><td>•</td><td> </td><td></td></tr></table><p>Now subtract the 5 hundredths.</p><table><tr><th>O</th><th>•</th><th>Tth</th><th>Hth</th></tr><tr><td></td><td>•</td><td> </td><td> </td></tr></table><p>Now subtract the 2 tenths, then the 2 ones.</p><table><tr><th>O</th><th>•</th><th>Tth</th><th>Hth</th></tr><tr><td> </td><td>•</td><td> </td><td> </td></tr></table><div><table><tr><td>O · Tth Hth</td></tr><tr><td>5 · 7 4</td></tr><tr><td>- 2 · 2 5</td></tr><tr><td>—</td></tr><tr><td>3 · 4 9</td></tr></table><table><tr><td>O · Tth Hth</td></tr><tr><td>5 · ⁶7 ¹4</td></tr><tr><td>- 2 · 2 5</td></tr><tr><td>—</td></tr><tr><td>3 · 4 9</td></tr></table><table><tr><td>O · Tth Hth</td></tr><tr><td>5 · ⁶7 ¹4</td></tr><tr><td>- 2 · 2 5</td></tr><tr><td>—</td></tr><tr><td>3 · 4 9</td></tr></table><table><tr><td>O · Tth Hth</td></tr><tr><td>5 · ⁶7 ¹4</td></tr><tr><td>- 2 · 2 5</td></tr><tr><td>—</td></tr><tr><td>3 · 4 9</td></tr></table></div></div>	O	•	Tth	Hth		•	 		O	•	Tth	Hth		•	 		O	•	Tth	Hth		•	 	 	O	•	Tth	Hth	 	•	 	 	O · Tth Hth	5 · 7 4	- 2 · 2 5	—	3 · 4 9	O · Tth Hth	5 · ⁶ 7 ¹ 4	- 2 · 2 5	—	3 · 4 9	O · Tth Hth	5 · ⁶ 7 ¹ 4	- 2 · 2 5	—	3 · 4 9	O · Tth Hth	5 · ⁶ 7 ¹ 4	- 2 · 2 5	—	3 · 4 9	<p>Use column subtraction, with an understanding of place value, including subtracting numbers with different numbers of decimal places.</p> <p>$3.921 - 3.75 = ?$</p> <div><table><tr><td>O · Tth Hth Thth</td></tr><tr><td>3 · 9 2 1</td></tr><tr><td>- 3 · 7 5 0</td></tr><tr><td>—</td></tr><tr><td>0 · 1 7 1</td></tr></table></div>	O · Tth Hth Thth	3 · 9 2 1	- 3 · 7 5 0	—	0 · 1 7 1
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Year 5 Multiplication															
Understanding factors	<p>Use cubes or counters to explore the meaning of 'square numbers'.</p> <p><i>25 is a square number because it is made from 5 rows of 5.</i></p> <p>Use cubes to explore cube numbers.</p>  <p><i>8 is a cube number.</i></p>	<p>Use images to explore examples and non-examples of square numbers.</p>  <p>$8 \times 8 = 64$ $8^2 = 64$</p>  <p><i>12 is not a square number, because you cannot multiply a whole number by itself to make 12.</i></p>	<p>Understand the pattern of square numbers in the multiplication tables.</p> <p>Use a multiplication grid to circle each square number. Can children spot a pattern?</p>												
Multiplying by 10, 100 and 1,000	<p>Use place value equipment to multiply by 10, 100 and 1,000 by unitising.</p> <table><tr><td>$4 \times 1 = 4 \text{ ones} = 4$</td><td></td></tr><tr><td>$4 \times 10 = 4 \text{ tens} = 40$</td><td></td></tr><tr><td>$4 \times 100 = 4 \text{ hundreds} = 400$</td><td></td></tr></table>	$4 \times 1 = 4 \text{ ones} = 4$		$4 \times 10 = 4 \text{ tens} = 40$		$4 \times 100 = 4 \text{ hundreds} = 400$		<p>Understand the effect of repeated multiplication by 10.</p> 	<p>Understand how exchange relates to the digits when multiplying by 10, 100 and 1,000.</p> <table><tr><td>H</td><td>T</td><td>O</td></tr><tr><td></td><td>I</td><td>7</td></tr></table> <p>$17 \times 10 = 170$ $17 \times 100 = 17 \times 10 \times 10 = 1,700$ $17 \times 1,000 = 17 \times 10 \times 10 \times 10 = 17,000$</p>	H	T	O		I	7
$4 \times 1 = 4 \text{ ones} = 4$															
$4 \times 10 = 4 \text{ tens} = 40$															
$4 \times 100 = 4 \text{ hundreds} = 400$															
H	T	O													
	I	7													

Multiplying by multiples of 10, 100 and 1,000

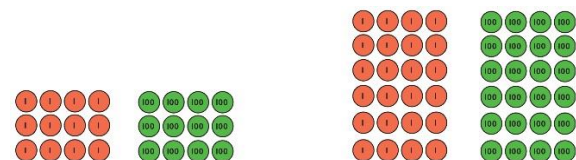
Use place value equipment to explore multiplying by unitising.



5 groups of 3 ones is 15 ones.
5 groups of 3 tens is 15 tens.

So, I know that 5 groups of 3 thousands would be 15 thousands.

Use place value equipment to represent how to multiply by multiples of 10, 100 and 1,000.



$$4 \times 3 = 12$$

$$4 \times 300 = 1,200$$

$$6 \times 4 = 24$$

$$6 \times 400 = 2,400$$

Use known facts and unitising to multiply.

$$5 \times 4 = 20$$

$$5 \times 40 = 200$$

$$5 \times 400 = 2,000$$

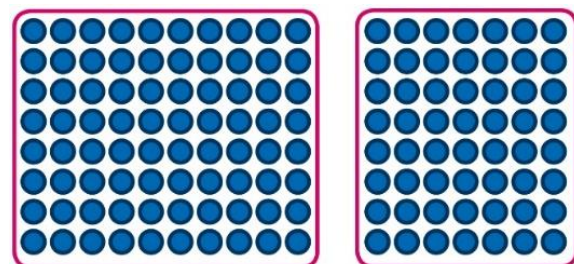
$$5 \times 4,000 = 20,000$$

$$5,000 \times 4 = 20,000$$

Multiplying up to 4-digit numbers by a single digit

Explore how to use partitioning to multiply efficiently.

$$8 \times 17 = ?$$



$$8 \times 10 = 80$$

$$8 \times 7 = 56$$

$$80 + 56 = 136$$

$$\text{So, } 8 \times 17 = 136$$

Represent multiplications using place value equipment and add the 1s, then 10s, then 100s, then 1,000s.

H	T	O
1000	10 10 10 10 10 10 10 10	1 1 1
1000	10 10 10 10 10 10 10 10	1 1 1
1000	10 10 10 10 10 10 10 10	1 1 1
1000	10 10 10 10 10 10 10 10	1 1 1
1000	10 10 10 10 10 10 10 10	1 1 1

Use an area model and then add the parts.

100	60	3
5 100 × 5 = 500	60 × 5 = 300	3 × 5 = 15

Use a column multiplication, including any required exchanges.

$$\begin{array}{r} 136 \\ \times 5 \\ \hline 816 \\ 23 \end{array}$$

Multiplying 2-digit numbers by 2-digit numbers

Partition one number into 10s and 1s, then add the parts.

$$23 \times 15 = ?$$



$$10 \times 15 = 150$$



$$10 \times 15 = 150$$



$$3 \times 15 = 45$$

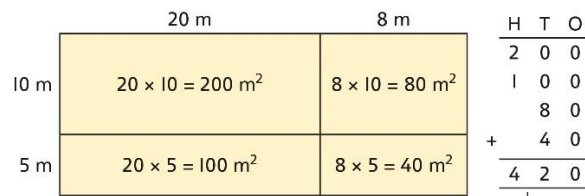
There are 345 bottles of milk in total.

H	T	O
1	5	0
1	5	0
+	4	5
3	4	5

$$23 \times 15 = 345$$

Use an area model and add the parts.

$$28 \times 15 = ?$$



$$28 \times 15 = 420$$

Use column multiplication, ensuring understanding of place value at each stage.

	3	4	
x	2	7	
	2	3	8

$$34 \times 7$$

$$200 + 80 + 40 = 320$$

$$34 \times 7 = 238$$

$$34 \times 20 = 680$$

$$238 + 680 = 918$$

$$34 \times 27 = 918$$

$$34 \times 20 = 680$$

$$34 \times 7 = 238$$

$$34 \times 27 = 918$$

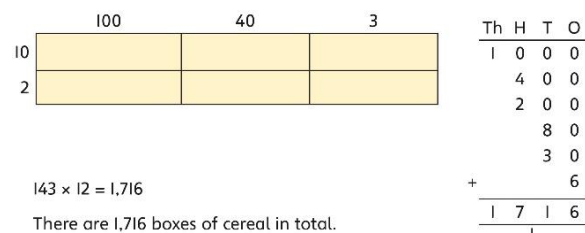
$$34 \times 20 = 680$$

$$34 \times 7 = 238$$

$$34 \times 27 = 918$$

Multiplying up to 4-digits by 2-digits

Use the area model then add the parts.



$$143 \times 12 = 1,716$$

There are 1,716 boxes of cereal in total.

$$143 \times 12 = 1,716$$

Use column multiplication, ensuring understanding of place value at each stage.

	1	4	3	
x		1	2	
	2	8	6	
	1	4	3	0
	1	7	1	6

$$143 \times 2$$

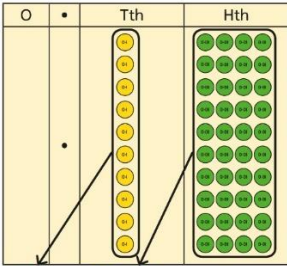
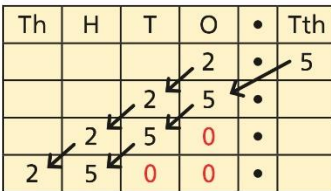
$$143 \times 10$$


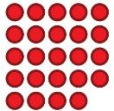
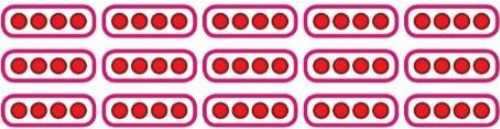
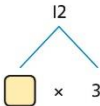
$$143 \times 12$$

Progress to include examples that require multiple exchanges as understanding, confidence and fluency build.

$$1,274 \times 32 = ?$$

First multiply 1,274 by 2.

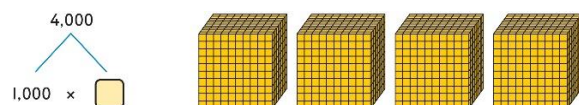
			$\begin{array}{r} 1\ 2\ 7\ 4 \\ \times \quad 3\ 2 \\ \hline 2\ 5\ 4\ 8 \\ 3\ 8\ 2\ 0 \\ \hline \end{array}$ <p>$1,274 \times 2$</p> <p>$1,274 \times 30$</p> <p><i>Then multiply 1,274 by 30.</i></p> $\begin{array}{r} 1\ 2\ 7\ 4 \\ \times \quad 3\ 2 \\ \hline 2\ 5\ 4\ 8 \\ 3\ 8\ 2\ 0 \\ \hline \end{array}$ <p>$1,274 \times 2$</p> <p>$1,274 \times 30$</p> <p><i>Finally, find the total.</i></p> $\begin{array}{r} 1\ 2\ 7\ 4 \\ \times \quad 3\ 2 \\ \hline 2\ 5\ 4\ 8 \\ 3\ 8\ 2\ 0 \\ \hline 4\ 0\ 7\ 6\ 8 \end{array}$ <p>$1,274 \times 32 = 40,768$</p>
<p>Multiplying decimals by 10, 100 and 1,000</p>	<p>Use place value equipment to explore and understand the exchange of 10 tenths, 10 hundredths or 10 thousandths.</p>	<p>Represent multiplication by 10 as exchange on a place value grid.</p>  <p>$0.14 \times 10 = 1.4$</p>	<p>Understand how this exchange is represented on a place value chart.</p>  <p>$2.5 \times 10 = 25$</p> <p>$2.5 \times 100 = 250$</p> <p>$2.5 \times 1,000 = 2,500$</p>

Year 5 Division			
Understanding factors and prime numbers	<p>Use equipment to explore the factors of a given number.</p>  $24 \div 3 = 8$ $24 \div 8 = 3$ <p>8 and 3 are factors of 24 because they divide 24 exactly.</p> $24 \div 5 = 4 \text{ remainder } 4.$  <p>5 is not a factor of 24 because there is a remainder.</p>	<p>Understand that prime numbers are numbers with exactly two factors.</p> $13 \div 1 = 13$ $13 \div 2 = 6 \text{ r } 1$ $13 \div 4 = 4 \text{ r } 1$ <p>1 and 13 are the only factors of 13. 13 is a prime number.</p>	<p>Understand how to recognise prime and composite numbers.</p> <p><i>I know that 31 is a prime number because it can be divided by only 1 and itself without leaving a remainder.</i></p> <p><i>I know that 33 is not a prime number as it can be divided by 1, 3, 11 and 33.</i></p> <p><i>I know that 1 is not a prime number, as it has only 1 factor.</i></p>
Understanding inverse operations and the link with multiplication, grouping and sharing	<p>Use equipment to group and share and to explore the calculations that are present.</p> <p><i>I have 28 counters.</i></p> <p><i>I made 7 groups of 4. There are 28 in total.</i></p> <p><i>I have 28 in total. I shared them equally into 7 groups. There are 4 in each group.</i></p> <p><i>I have 28 in total. I made groups of 4. There are 7 equal groups.</i></p>	<p>Represent multiplicative relationships and explore the families of division facts.</p>  $60 \div 4 = 15$ $60 \div 15 = 4$	<p>Represent the different multiplicative relationships to solve problems requiring inverse operations.</p> $12 \div 3 = \square$ $12 \div \square = 3$ $\square \times 3 = 12$ $\square \div 3 = 12$  <p>Understand missing number problems for division calculations and know how to solve them using inverse operations.</p> $22 \div ? = 2$ $22 \div 2 = ?$ $? \div 2 = 22$ $? \div 22 = 2$

Dividing whole numbers by 10, 100 and 1,000

Use place value equipment to support unitising for division.

$$4,000 \div 1,000$$



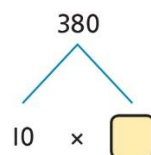
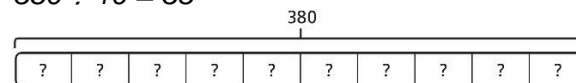
4,000 is 4 thousands.

$$4 \times 1,000 = 4,000$$

$$\text{So, } 4,000 \div 1,000 = 4$$

Use a bar model to support dividing by unitising.

$$380 \div 10 = 38$$



380 is 38 tens.

$$38 \times 10 = 380$$

$$10 \times 38 = 380$$

$$\text{So, } 380 \div 10 = 38$$

Understand how and why the digits change on a place value grid when dividing by 10, 100 or 1,000.

Th	H	T	O
3	2	0	0

$$3,200 \div 100 = ?$$

3,200 is 3 thousands and 2 hundreds.

$$200 \div 100 = 2$$

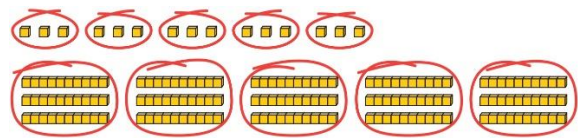
$$3,000 \div 100 = 30$$

$$3,200 \div 100 = 32$$

So, the digits will move two places to the right.

Dividing by multiples of 10, 100 and 1,000

Use place value equipment to represent known facts and unitising.



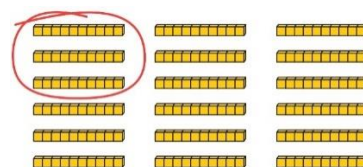
15 ones put into groups of 3 ones. There are 5 groups.

$$15 \div 3 = 5$$

15 tens put into groups of 3 tens. There are 5 groups.

$$150 \div 30 = 5$$

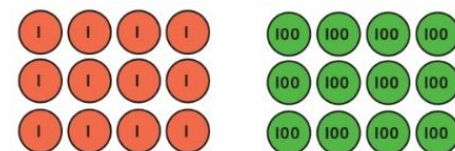
Represent related facts with place value equipment when dividing by unitising.



180 is 18 tens.

18 tens divided into groups of 3 tens. There are 6 groups.

$$180 \div 30 = 6$$



Reason from known facts, based on understanding of unitising. Use knowledge of the inverse relationship to check.

$$3,000 \div 5 = 600$$

$$3,000 \div 50 = 60$$


$$3,000 \div 500 = 6$$

$$5 \times 600 = 3,000$$

$$50 \times 60 = 3,000$$

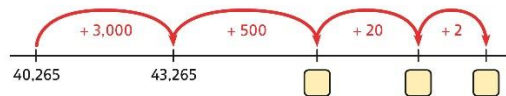
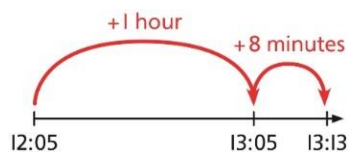
$$500 \times 6 = 3,000$$


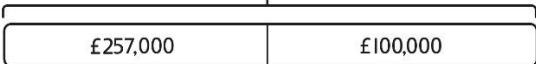
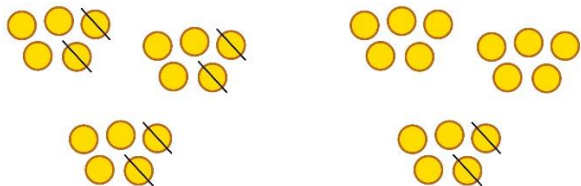
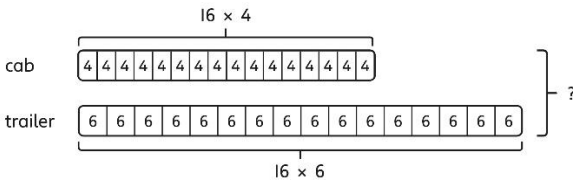
		<p>12 ones divided into groups of 4. There are 3 groups.</p> <p>12 hundreds divided into groups of 4 hundreds. There are 3 groups.</p> <p>$1200 \div 400 = 3$</p>	
<p>Dividing up to four digits by a single digit using short division</p>	<p>Explore grouping using place value equipment.</p> <p>$268 \div 2 = ?$</p> <p><i>There is 1 group of 2 hundreds. There are 3 groups of 2 tens. There are 4 groups of 2 ones.</i></p> <p>$264 \div 2 = 134$</p>	<p>Use place value equipment on a place value grid alongside short division. The model uses grouping. A sharing model can also be used, although the model would need adapting.</p> <div style="display: flex; align-items: center;"> <div style="margin-right: 10px;"> $\begin{array}{r} 4 \overline{) 48} \\ 4 \\ \hline \end{array}$ $\begin{array}{r} 1 \\ 4 \overline{) 48} \\ 4 \\ \hline \end{array}$ $\begin{array}{r} 12 \\ 4 \overline{) 48} \\ 4 \\ \hline \end{array}$ </div> <div style="display: flex; flex-direction: column; align-items: center;"> <div style="display: flex; justify-content: space-around; width: 100%;"> <div style="border: 1px solid black; padding: 5px; text-align: center;">T</div> <div style="border: 1px solid black; padding: 5px; text-align: center;">O</div> </div> <div style="display: flex; justify-content: space-around; width: 100%;"> <div style="border: 1px solid black; padding: 5px; text-align: center;">T</div> <div style="border: 1px solid black; padding: 5px; text-align: center;">O</div> </div> <div style="display: flex; justify-content: space-around; width: 100%;"> <div style="border: 1px solid black; padding: 5px; text-align: center;">T</div> <div style="border: 1px solid black; padding: 5px; text-align: center;">O</div> </div> </div> </div>	<p>Use short division for up to 4-digit numbers divided by a single digit.</p> $\begin{array}{r} 0 \ 5 \ 5 \ 6 \\ 7 \overline{) 3 \ 8 \ 9 \ 2} \end{array}$ <p>$3,892 \div 7 = 556$</p> <p>Use multiplication to check.</p> <p>$556 \times 7 = ?$</p> <p>$6 \times 7 = 42$ $50 \times 7 = 350$ $500 \times 7 = 3500$</p> <p>$3,500 + 350 + 42 = 3,892$</p>

		<div> $\begin{array}{r} 4 \overline{) 92} \\ 8 \\ \hline 12 \\ 12 \\ \hline 0 \end{array}$ <p>First, lay out the problem.</p> </div> <div> $\begin{array}{r} 2 \\ 4 \overline{) 92} \\ 8 \\ \hline 12 \\ 12 \\ \hline 0 \end{array}$ <p>How many groups of 4 go into 9 tens?</p> <p>2 groups of 4 tens with 1 ten left over.</p> </div> <div> $\begin{array}{r} 2 \\ 4 \overline{) 92} \\ 8 \\ \hline 12 \\ 12 \\ \hline 0 \end{array}$ <p>Exchange the 1 ten left over for 10 ones.</p> <p>We now have 12 ones.</p> </div> <div> $\begin{array}{r} 2 3 \\ 4 \overline{) 92} \\ 8 \\ \hline 12 \\ 12 \\ \hline 0 \end{array}$ <p>How many groups of 4 go into 12 ones?</p> <p>3 groups of 4 ones.</p> </div>	
<p>Understanding remainders</p>	<p>Understand remainders using concrete versions of a problem.</p> <p><i>80 cakes divided into trays of 6.</i></p>  <p><i>80 cakes in total. They make 13 groups of 6, with 2 remaining.</i></p>	<p>Use short division and understand remainders as the last remaining 1s.</p> <div> $\begin{array}{r} 6 \overline{) 80} \\ 6 \\ \hline 20 \end{array}$ <p>Lay out the problem as short division.</p> </div> <div> $\begin{array}{r} 1 \\ 6 \overline{) 80} \\ 6 \\ \hline 20 \end{array}$ <p>How many groups of 6 go into 8 tens?</p> <p>There is 1 group of 6 tens.</p> <p>There are 2 tens remaining.</p> </div> <div> $\begin{array}{r} 1 3 \text{ r } 2 \\ 6 \overline{) 80} \\ 6 \\ \hline 20 \end{array}$ <p>How many groups of 6 go into 20 ones?</p> <p>There are 3 groups of 6 ones.</p> <p>There are 2 ones remaining.</p> </div>	<p>In problem solving contexts, represent divisions including remainders with a bar model.</p> <div> $\begin{array}{c} 683 \\ \hline 136 \quad 136 \quad 136 \quad 136 \quad 136 \quad 3 \end{array}$ </div> <p>$683 = 136 \times 5 + 3$</p> <p>$683 \div 5 = 136 \text{ r } 3$</p>

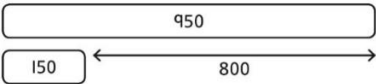
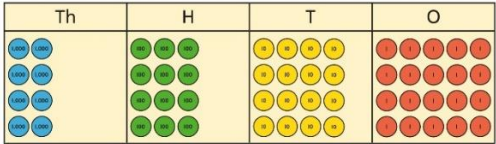
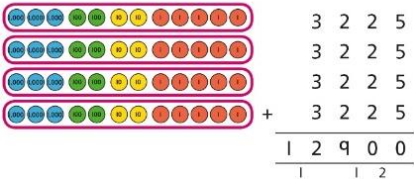
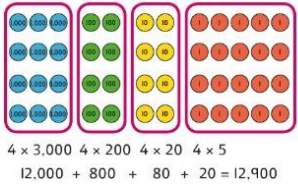
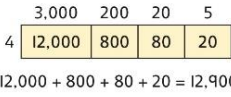

Dividing decimals by 10, 100 and 1,000	<p>Understand division by 10 using exchange.</p> <p>2 ones are 20 tenths.</p> <p>20 tenths divided by 10 is 2 tenths.</p>	<p>Represent division using exchange on a place value grid.</p> <table><tr><td>O</td><td>•</td><td>Tth</td><td>Hth</td></tr><tr><td>1</td><td>•</td><td>0</td><td>0</td></tr></table> <table><tr><td>O</td><td>•</td><td>Tth</td><td>Hth</td></tr><tr><td>1</td><td>•</td><td>0</td><td>0</td></tr><tr><td></td><td>•</td><td>10</td><td>0</td></tr><tr><td></td><td>•</td><td>0</td><td>50</td></tr></table> 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<p>Selecting mental methods for larger numbers where appropriate</p>	<p>Represent 7-digit numbers on a place value grid, and use this to support thinking and mental methods.</p>  <p>$2,411,301 + 500,000 = ?$</p> <p><i>This would be 5 more counters in the HTh place.</i></p> <p><i>So, the total is 2,911,301.</i></p> <p>$2,411,301 + 500,000 = 2,911,301$</p>	<p>Use a bar model to support thinking in addition problems.</p> <p>$257,000 + 99,000 = ?$</p>  <p><i>I added 100 thousands then subtracted 1 thousand.</i></p> <p>$257 \text{ thousands} + 100 \text{ thousands} = 357 \text{ thousands}$</p> <p>$257,000 + 100,000 = 357,000$ $357,000 - 1,000 = 356,000$</p> <p><i>So, $257,000 + 99,000 = 356,000$</i></p>	<p>Use place value and unitising to support mental calculations with larger numbers.</p> <p>$195,000 + 6,000 = ?$</p> <p>$195 + 5 + 1 = 201$</p> <p>$195 \text{ thousands} + 6 \text{ thousands} = 201 \text{ thousands}$</p> <p><i>So, $195,000 + 6,000 = 201,000$</i></p>
<p>Understanding order of operations in calculations</p>	<p>Use equipment to model different interpretations of a calculation with more than one operation. Explore different results.</p> <p>$3 \times 5 - 2 = ?$</p>  <p> $\begin{array}{r} 3 \times 5 - 2 \\ \downarrow \downarrow \\ 3 \times 3 = 9 \end{array}$ </p> <p> $\begin{array}{r} 3 \times 5 - 2 \\ \downarrow \downarrow \\ 15 - 2 = 13 \end{array}$ </p>	<p>Model calculations using a bar model to demonstrate the correct order of operations in multi-step calculations.</p>  <p>This can be written as: $16 \times 4 + 16 \times 6$</p> <p>$16 \times 4 + 16 \times 6$ $64 + 96 = 160$</p>	<p>Understand the correct order of operations in calculations without brackets.</p> <p>Understand how brackets affect the order of operations in a calculation.</p> <p>$4 + 6 \times 16$ $4 + 96 = 100$</p> <p>$(4 + 6) \times 16$ $10 \times 16 = 160$</p>

<div>Year 6</div> <div>Subtraction</div>																																																																							
<div>Comparing and selecting efficient methods</div>	<div>Use counters on a place value grid to represent subtractions of larger numbers.</div> <div><table><tr><th>Th</th><th>H</th><th>T</th><th>O</th></tr><tr><td><div><div></div><div></div></div></td><td><div><div></div><div></div><div></div><div></div><div></div><div></div><div></div><div></div></div></td><td><div><div></div><div></div><div></div><div></div><div></div><div></div><div></div><div></div></div></td><td><div><div></div><div></div><div></div><div></div><div></div><div></div><div></div><div></div></div></td></tr></table></div>	Th	H	T	O	<div><div></div><div></div></div>	<div><div></div><div></div><div></div><div></div><div></div><div></div><div></div><div></div></div>	<div><div></div><div></div><div></div><div></div><div></div><div></div><div></div><div></div></div>	<div><div></div><div></div><div></div><div></div><div></div><div></div><div></div><div></div></div>	<div>Compare subtraction methods alongside place value representations.</div> <div><div><div><div></div><div></div><div></div><div></div></div><div><div></div><div></div><div></div><div></div></div><div><div></div><div></div><div></div><div></div></div><div><div></div><div></div><div></div><div></div></div></div><div><div></div><div></div><div></div><div></div></div><div><div></div><div></div><div></div><div></div></div><div><div></div><div></div><div></div><div></div></div><div><div></div><div></div><div></div><div></div></div></div> <div><div></div><div></div><div></div><div></div></div> <div><div></div><div></div><div></div><div></div></div> <div><div></div><div></div><div></div><div></div></div> <div><div></div><div></div><div></div><div></div></div> <div><table><tr><th>Th</th><th>H</th><th>T</th><th>O</th></tr><tr><td><div><div></div><div></div></div></td><td><div><div></div><div></div><div></div><div></div><div></div><div></div><div></div><div></div></div></td><td><div><div></div><div></div><div></div><div></div><div></div><div></div><div></div><div></div></div></td><td><div><div></div><div></div><div></div><div></div><div></div><div></div><div></div><div></div></div></td></tr></table><div><table><tr><th>Th</th><th>H</th><th>T</th><th>O</th></tr><tr><td>2</td><td>6</td><td>7</td><td>9</td></tr><tr><td>-</td><td>5</td><td>3</td><td>4</td></tr><tr><td>2</td><td>1</td><td>4</td><td>5</td></tr></table></div></div>	Th	H	T	O	<div><div></div><div></div></div>	<div><div></div><div></div><div></div><div></div><div></div><div></div><div></div><div></div></div>	<div><div></div><div></div><div></div><div></div><div></div><div></div><div></div><div></div></div>	<div><div></div><div></div><div></div><div></div><div></div><div></div><div></div><div></div></div>	Th	H	T	O	2	6	7	9	-	5	3	4	2	1	4	5	<div>Compare and select methods.</div> <div>Use column subtraction when mental methods are not efficient.</div> <div>Use two different methods for one calculation as a checking strategy.</div> <div><table><tr><th>Th</th><th>H</th><th>T</th><th>O</th></tr><tr><td>1</td><td>8</td><td>14</td><td>2</td></tr><tr><td>-</td><td>1</td><td>5</td><td>5</td></tr><tr><td>3</td><td>9</td><td>4</td><td></td></tr></table><div><div><div></div><div></div><div></div><div></div></div><div><div></div><div></div><div></div><div></div></div><div><div></div><div></div><div></div><div></div></div><div><div></div><div></div><div></div><div></div></div></div><div><div></div><div></div><div></div><div></div></div><div><div></div><div></div><div></div><div></div></div><div><div></div><div></div><div></div><div></div></div><div><div></div><div></div><div></div><div></div></div></div> <div>Use column subtraction for decimal problems, including in the context of measure.</div> <div><table><tr><th>H</th><th>T</th><th>O</th><th>Tth</th><th>Hth</th></tr><tr><td>3</td><td>0</td><td>9</td><td>6</td><td>0</td></tr><tr><td>-</td><td>2</td><td>0</td><td>6</td><td>4</td></tr><tr><td>1</td><td>0</td><td>3</td><td>2</td><td>0</td></tr></table></div>	Th	H	T	O	1	8	14	2	-	1	5	5	3	9	4		H	T	O	Tth	Hth	3	0	9	6	0	-	2	0	6	4	1	0	3	2	0
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<p>Subtracting mentally with larger numbers</p>		<p>Use a bar model to show how unitising can support mental calculations.</p> <p>$950,000 - 150,000$ That is 950 thousands – 150 thousands</p>  <p>So, the difference is 800 thousands. $950,000 - 150,000 = 800,000$</p>	<p>Subtract efficiently from powers of 10.</p> <p>$10,000 - 500 = ?$</p>
<p>Year 6 Multiplication</p>			
<p>Multiplying up to a 4-digit number by a single digit number</p>	<p>Use equipment to explore multiplications.</p>  <p>4 groups of 2,345</p> <p>This is a multiplication:</p> <p>$4 \times 2,345$ $2,345 \times 4$</p>	<p>Use place value equipment to compare methods.</p> <p>Method 1</p>  <p>Method 2</p> 	<p>Understand area model and short multiplication.</p> <p>Compare and select appropriate methods for specific multiplications.</p> <p>Method 3</p>  <p>$12,000 + 800 + 80 + 20 = 12,900$</p> <p>Method 4</p> 

Multiplying up to a 4-digit number by a 2-digit number

Use an area model alongside written multiplication.

Method 1

	1,000	200	30	5
20	20,000	4,000	600	100
1	1,000	200	30	5

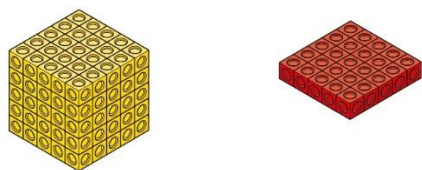
	1	2	3	5
×		2	1	
				5
			3	0
		2	0	0
	1	0	0	0
	1	0	0	0
	6	0	0	0
	4	0	0	0
	2	0	0	0
	2	5	9	3

Use compact column multiplication with understanding of place value at all stages.

	1	2	3	5	
×		2	1		
					1 × 1,235
	2	4	7	0	0
	2	5	9	3	5

Using knowledge of factors and partitions to compare methods for multiplications

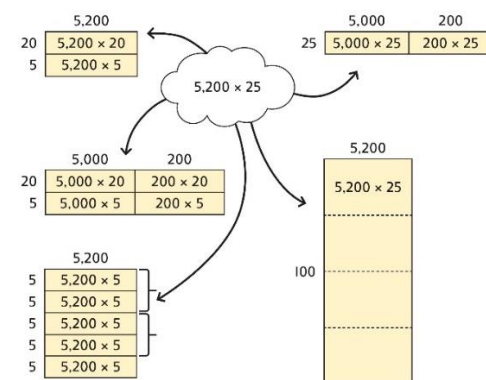
Use equipment to understand square numbers and cube numbers.



$$5 \times 5 = 5^2 = 25$$

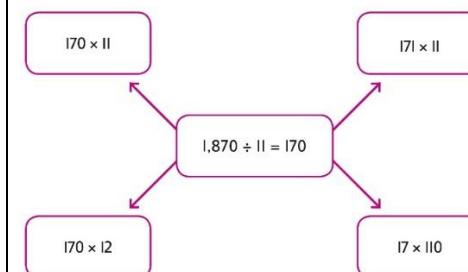
$$5 \times 5 \times 5 = 5^3 = 25 \times 5 = 125$$

Compare methods visually using an area model. Understand that multiple approaches will produce the same answer if completed accurately.



Represent and compare methods using a bar model.

Use a known fact to generate families of related facts.



Use factors to calculate efficiently.

$$15 \times 16$$

$$= 3 \times 5 \times 2 \times 8$$

$$= 3 \times 8 \times 2 \times 5$$

$$= 24 \times 10$$

$$= 240$$

Multiplying by 10, 100 and 1,000

Use place value equipment to explore exchange in decimal multiplication.

T	O	.	Tth
		.	3

Represent 0.3.

T	O	.	Tth
		.	30

Multiply by 10.

T	O	.	Tth
3		.	0

Exchange each group of ten tenths.

$$0.3 \times 10 = ?$$

0.3 is 3 tenths.

10 \times 3 tenths are 30 tenths.

30 tenths are equivalent to 3 ones.

Understand how the exchange affects decimal numbers on a place value grid.

T	O	.	Tth
3		.	0

T	O	.	Tth
	3	.	0

$$0.3 \times 10 = 3$$

Use knowledge of multiplying by 10, 100 and 1,000 to multiply by multiples of 10, 100 and 1,000.

$$8 \times 100 = 800$$

$$8 \times 300 = 800 \times 3 = 2,400$$

$$2.5 \times 10 = 25$$

$$2.5 \times 20 = 2.5 \times 10 \times 2 = 50$$

Multiplying decimals

Explore decimal multiplications using place value equipment and in the context of measures.

0.1	0.1	0.1	0.1
0.1	0.1	0.1	0.1
0.1	0.1	0.1	0.1

3 groups of 4 tenths is 12 tenths.

4 groups of 3 tenths is 12 tenths.



1.3 cm 1.3 cm 1.3 cm 1.3 cm

$$4 \times 1 \text{ cm} = 4 \text{ cm}$$

$$4 \times 0.3 \text{ cm} = 1.2 \text{ cm}$$

$$4 \times 1.3 = 4 + 1.2 = 5.2 \text{ cm}$$

Represent calculations on a place value grid.

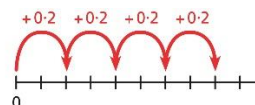
$$3 \times 3 = 9$$

$$3 \times 0.3 = 0.9$$

T	O	.	Tth
		.	9

Understand the link between multiplying decimals and repeated addition.

T	O	.	Tth
		.	9



Use known facts to multiply decimals.

$$4 \times 3 = 12$$

$$4 \times 0.3 = 1.2$$

$$4 \times 0.03 = 0.12$$

$$20 \times 5 = 100$$

$$20 \times 0.5 = 10$$

$$20 \times 0.05 = 1$$

Find families of facts from a known multiplication.

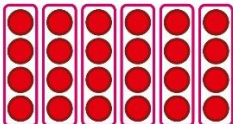
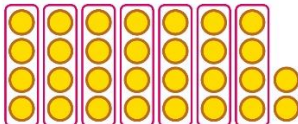
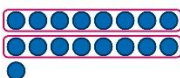
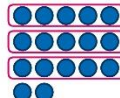
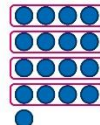

I know that $18 \times 4 = 72$.

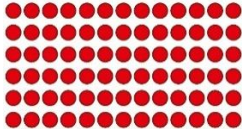
This can help me work out:

$$1.8 \times 4 = ?$$

$$18 \times 0.4 = ?$$

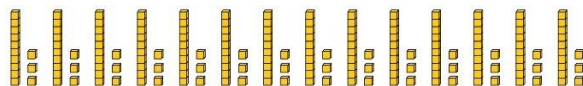
$$180 \times 0.4 = ?$$

			<div>$18 \times 0.04 = ?$</div> <div>Use a place value grid to understand the effects of multiplying decimals.</div> <div><table><tr><th></th><th>H</th><th>T</th><th>O</th><th>•</th><th>Tth</th><th>Hth</th></tr><tr><td>2×3</td><td></td><td></td><td>6</td><td>•</td><td></td><td></td></tr><tr><td>0.2×3</td><td></td><td></td><td>0</td><td>•</td><td>6</td><td></td></tr><tr><td>0.02×3</td><td></td><td></td><td></td><td>•</td><td></td><td></td></tr></table></div>		H	T	O	•	Tth	Hth	2×3			6	•			0.2×3			0	•	6		0.02×3				•																								
	H	T	O	•	Tth	Hth																																															
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<div>Year 6 Division</div>																																																					
<div>Understanding factors</div>	<div>Use equipment to explore different factors of a number.</div> <div><div><div>$24 \div 4 = 6$</div></div><div><div>$30 \div 4 = 7 \text{ remainder } 2$</div></div></div> <div><i>4 is a factor of 24 but is not a factor of 30.</i></div>	<div>Recognise prime numbers as numbers having exactly two factors. Understand the link with division and remainders.</div> <div><div><div>$17 \div 2 = 8 \text{ r } 1$</div></div><div><div>$17 \div 3 = 5 \text{ r } 2$</div></div><div><div>$17 \div 4 = 4 \text{ r } 1$</div></div><div><div>$17 \div 5 = 3 \text{ r } 2$</div></div></div>	<div>Recognise and know primes up to 100. Understand that 2 is the only even prime, and that 1 is not a prime number.</div> <div><table><tr><td>1</td><td>2</td><td>3</td><td>4</td><td>5</td><td>6</td><td>7</td><td>8</td><td>9</td><td>10</td></tr><tr><td>11</td><td>12</td><td>13</td><td>14</td><td>15</td><td>16</td><td>17</td><td>18</td><td>19</td><td>20</td></tr><tr><td>21</td><td>22</td><td>23</td><td>24</td><td>25</td><td>26</td><td>27</td><td>28</td><td>29</td><td>30</td></tr><tr><td>31</td><td>32</td><td>33</td><td>34</td><td>35</td><td>36</td><td>37</td><td>38</td><td>39</td><td>40</td></tr><tr><td>41</td><td>42</td><td>43</td><td>44</td><td>45</td><td>46</td><td>47</td><td>48</td><td>49</td><td>50</td></tr></table></div>	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	21	22	23	24	25	26	27	28	29	30	31	32	33	34	35	36	37	38	39	40	41	42	43	44	45	46	47	48	49	50
1	2	3	4	5	6	7	8	9	10																																												
11	12	13	14	15	16	17	18	19	20																																												
21	22	23	24	25	26	27	28	29	30																																												
31	32	33	34	35	36	37	38	39	40																																												
41	42	43	44	45	46	47	48	49	50																																												

<div>Dividing by a single digit</div>	<div>Use equipment to make groups from a total.</div> <div></div> <div><div>There are 78 in total.</div><div>There are 6 groups of 13.</div><div>There are 13 groups of 6.</div></div>	<div><table><tr><td>H</td><td>T</td><td>O</td></tr><tr><td>●</td><td>●●●●●●</td><td>●●</td></tr></table><div>How many groups of 6 are in 100?</div><div>$6 \overline{) 1 \ 3 \ 2}$</div></div> <div><table><tr><td>H</td><td>T</td><td>O</td></tr><tr><td>●</td><td>●●●●●●</td><td>●●</td></tr></table><div>How many groups of 6 are in 13 tens?</div><div>$6 \overline{) 0 \ 2}$</div></div> <div><table><tr><td>H</td><td>T</td><td>O</td></tr><tr><td></td><td>●●●●●●</td><td>●●●●●●</td></tr></table><div>How many groups of 6 are in 12 ones?</div><div>$6 \overline{) 0 \ 2 \ 2}$</div></div>	H	T	O	●	●●●●●●	●●	H	T	O	●	●●●●●●	●●	H	T	O		●●●●●●	●●●●●●	<div>Use short division to divide by a single digit.</div> <div>$6 \overline{) 1 \ 3 \ 2}$</div> <div>$6 \overline{) 0 \ 2}$</div> <div>$6 \overline{) 0 \ 2 \ 2}$</div> <div><div>Use an area model to link multiplication and division.</div><div><div><div><div>?</div><div>132</div></div><div>$6 \times ? = 132$</div></div><div><table><tr><td>10</td><td>10</td><td>1</td><td>1</td></tr><tr><td>60</td><td>60</td><td>6</td><td>6</td></tr></table><div>$132 = 120 + 12$</div><div>$132 \div 6 = 20 + 2 = 22$</div></div></div></div>	10	10	1	1	60	60	6	6
H	T	O																											
●	●●●●●●	●●																											
H	T	O																											
●	●●●●●●	●●																											
H	T	O																											
	●●●●●●	●●●●●●																											
10	10	1	1																										
60	60	6	6																										
<div>Dividing by a 2-digit number using factors</div>	<div>Understand that division by factors can be used when dividing by a number that is not prime.</div>	<div>Use factors and repeated division.</div> <div><div><div>1,260</div><div><table><tr><td></td><td></td></tr><tr><td></td><td></td></tr></table></div></div><div>$1,260 \div 2 = 630$</div><div><div>$630 \div 7 = 90$</div><div>$1,260 \div 14 = 90$</div></div><div><table><tr><td></td><td></td><td></td><td></td><td></td><td></td></tr><tr><td></td><td></td><td></td><td></td><td></td><td></td></tr></table></div></div>																	<div>Use factors and repeated division where appropriate.</div> <div><div>$2,100 \div 12 = ?$</div><div><div><div>$2,100 \rightarrow \div 2 \rightarrow \div 6 \rightarrow$</div><div>$2,100 \rightarrow \div 6 \rightarrow \div 2 \rightarrow$</div><div>$2,100 \rightarrow \div 3 \rightarrow \div 4 \rightarrow$</div><div>$2,100 \rightarrow \div 4 \rightarrow \div 3 \rightarrow$</div><div>$2,100 \rightarrow \div 3 \rightarrow \div 2 \rightarrow \div 2 \rightarrow$</div></div></div></div>										

Dividing by a 2-digit number using long division

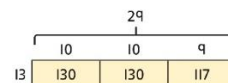
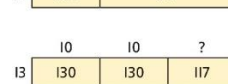
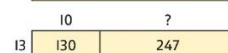
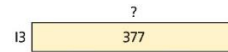
Use equipment to build numbers from groups.



182 divided into groups of 13.
There are 14 groups.

Use an area model alongside written division to model the process.

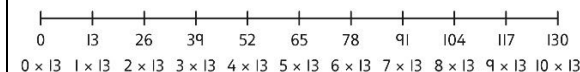
$$377 \div 13 = ?$$



$$377 \div 13 = 29$$

Use long division where factors are not useful (for example, when dividing by a 2-digit prime number).
Write the required multiples to support the division process.

$$377 \div 13 = ?$$



$$\begin{array}{r} 13 \overline{) 377} \\ - 130 \quad 10 \\ \hline 247 \\ - 130 \quad 10 \\ \hline 117 \\ - 117 \quad 9 \\ \hline 0 \end{array}$$

$$377 \div 13 = 29$$

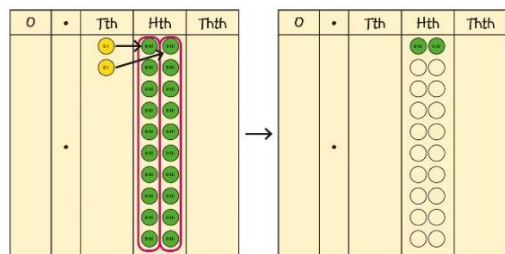
A slightly different layout may be used, with the division completed above rather than at the side.

$$\begin{array}{r} 3 \\ 21 \overline{) 798} \\ - 630 \\ \hline 168 \end{array}$$

$$\begin{array}{r} 38 \\ 21 \overline{) 798} \\ - 630 \\ \hline 168 \\ - 168 \\ \hline 0 \end{array}$$

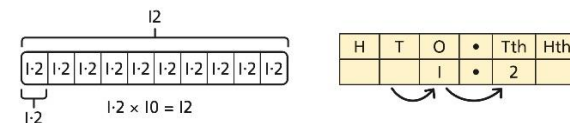
Dividing by 10, 100 and 1,000

Use place value equipment to explore division as exchange.

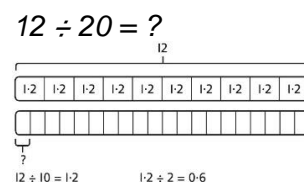


0.2 is 2 tenths.
2 tenths is equivalent to 20 hundredths.
20 hundredths divided by 10 is 2 hundredths.

Represent division to show the relationship with multiplication. Understand the effect of dividing by 10, 100 and 1,000 on the digits on a place value grid.

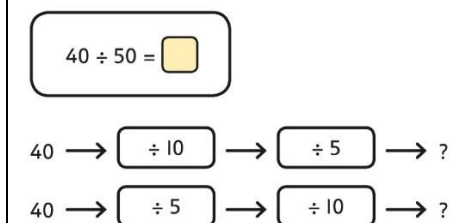


Understand how to divide using division by 10, 100 and 1,000.



Divisions with a remainder explored in problem-solving contexts.

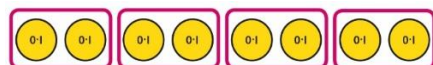
Use knowledge of factors to divide by multiples of 10, 100 and 1,000.



$40 \div 5 = 8$
 $8 \div 10 = 0.8$
So, $40 \div 50 = 0.8$

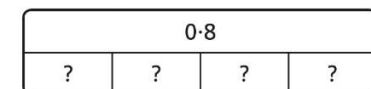
Dividing decimals

Use place value equipment to explore division of decimals.



8 tenths divided into 4 groups. 2 tenths in each group.

Use a bar model to represent divisions.



$4 \times 2 = 8$ $8 \div 4 = 2$
So, $4 \times 0.2 = 0.8$ $0.8 \div 4 = 0.2$

Use short division to divide decimals with up to 2 decimal places.

